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ITERATIVE NONLINEAR GOAL PROGRAMMING AND APPLICATION TO PRODUCTION PLANNING PROBLEMS

by

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CHAPTER 1

INTRODUCTION

The first objective of this report is to develop an algorithm for solving nonlinear multiple objective problems using goal programming.

There are several classes of methods which can handle multiobjective problems. Hwang et al. [8] discussed and classified these methods into various major classes according to the stage at which information is needed and the type of information that is needed. The following three are the most widely used analytical methods to solve multiple objective problems.

1. Interactive multiobjective programming [1, 13].

This technique allows the decision maker to trade off one objective versus another in an interactive manner. In this method a search procedure is used to find the prefered solution with questions being asked of the decision maker at each step of the search in order to determine a new estimate of the solution.

2. Multiobjective programming with utility function [11].

In this all the multiobjectives are reduced to a single objective function by using utility functions.

3. Goal programming [2,10,12,9].

In this technique, all of the decision maker's targets or goals may be incorporated into the achievement function. The objectives of the goal programming need not be of a single dimension. The set of physical conditions of the problem must be satisfied before any goal is considered. The set of feasible solutions which satisfies the physical conditions is established. The optimal solution then is selected from the feasible solution which best fulfills the decision maker's stated goals.

The weakness of the first two methods is that they are dependent upon the ability of the decision maker to conduct sequence of communications about his preference to the model. Hence the goal programming technique is an appropriate tool in solving the general multiple objective problems.

The initial work on goal programming was done by Charnes and Cooper [2] in 1961. The work of Charnes and Cooper, Ijiri [10], and others [12,9,3] resulted in a systematic methodology known as goal programming for solving multiobjective problems.

An iterative approach to solve goal programming problems was developed by Dauer and Krueger [3]. An algorithm, using a Hook and Jeeves pattern search, was presented by Ignizio [9] for nonlinear goal programming problems. A new algorithm, which integrates the iterative approach and the modified Hooke and Jeeves pattern search, is developed. The iterative approach

used in the new algorithm follows closely to that of Dauer and Krueger. In chapter 2, the new algorithm and its application is explained through a numerical example.

The second objective of this report is to apply nonlinear goal programming technique to production planning problems which have multiple objectives.

Aggregate production planning is extremely important for any firm to achieve the most efficient utilization of available resources while meeting the restrictions imposed by the environment as well by organizational policies concerning employment, inventories, production, and the use of outside capacity.

The traditional approach to production planning problems is to reduce multiple objectives into a single objective function which usually requires obscure cost information. Since no satisfactory method is available to determine costs objectively, the procedure usually followed is to ask management to provide its best estimate of costs. But it is hard to find managers who can provide concrete estimates of these costs. However, if we assume that the management can provide an ordinal measure of various objectives, goal programming can provide an improved model to solve the problem of aggregate production planning.

Lee 12 applied goal programming technique to solve production planning problems having linear multiple objectives.

However, it often may be the case that one or more objectives may be nonlinear in nature. Goodman [4] applied goal programming techniques to the Holt et al. [5] model by linearizing the cost terms. However, it is not always possible to linearize certain objectives. So a direct method, nonlinear goal programming, is used to solve such problems.

In Chapter 3, the production planning problem of Holt et al. is modified by adding two more objectives. The problem then is formulated as a goal programming model and is solved using the iterative nonlinear goal programming technique as discussed in Chapter 2. In Chapter 4, a general multiobjective aggregate production planning problem is formulated as a goal programming model. The solution is obtained by using the iterative nonlinear goal programming.

CHAPTER 2

NONLINEAR GOAL PROGRAMMING METHOD (NLGP) AN ITERATIVE APPROACH

The general mathematical representation of the multiple objective decision problem is

$$\left.\begin{array}{l} \text{Max } [f_1(X),\ f_2(X),\ \ldots,\ f_k(X)] \\ \text{subject to} \quad h_i(X) \leq c_i, \quad i = 1, \ldots, m \end{array}\right\} \quad (1)$$

where X is a n-dimensional decision variable vector. The problem has k objectives which are to be maximized, m constraints and n decision variables. Any or all of the functions may be nonlinear. In the goal programming (GP) approach of solving the problem as posed by (1), the decision maker (DM) is required to indicate his target or goals for each of the k objectives. Let these goals be b_i, $i = 1, \ldots, k$. The DM is also required to indicate the relative importance of the achievement of these goals by giving an ordinal ranking of the goals. More than one objective can be in one single ranking provided their units are commensurable.

In the mathematical formulation of the problem, deviational variables are attached (one negative and one positive deviational variable) to each of the objective function equations and constraint equations. Thus, the new converted problem has two sets of equality constraints: one set is called the 'absolute constraints' formed from the original

problem constraints, the other set is called 'goal constraints' formed from the objective functions. So, for the problem given in (1), the GP constraint set is given by (2):

Absolute constraints:

$$h_i(X) + d_i^- - d_i^+ = c_i, \quad i = 1, \ldots, m$$

Goal constraints: (2)

$$f_i(X) + d_{m+i}^- - d_{m+i}^+ = b_i, \; i = 1, \ldots, k$$

The negative deviational variable, d_i^-, indicates the under-achievement of c_i or b_i; the positive deviational variable, d_i^+, indicates the overachievement; and $d^-, d^+ \geq 0$, $d_i^- \cdot d_i^+ = 0$.

The next step of the formulation is to form the achievement functions. There is one achievement function for each of the priority level of ranking of the goal as indicated by the DM. These achievement functions are linear functions of proper deviational variables for a particular level of ranking. The complete achievement function is shown as

$$\text{Min:} \quad \bar{a} = [P_1 a_1(d^-, d^+), \; P_2 a_2(d^-, d^+), \; \ldots, \; P_\ell a_\ell(d^-, d^+)] \quad (3)$$

where the preemptive priority weights, P_i's, are such that no number W, however, large it is, can make $W.P_{j+1} > P_j$; that is, $P_1 >>> P_2 >>> \ldots >>> P_\ell$. It must be noted that the first priority level P_1 is associated with the achievement of the absolute constraints; i.e., $a_1(d_i^-, d_i^+)$, where i = 1, ..., m.

The minimization of $\bar{a}$ is done iteratively starting with a_1. Next a_2 is minimized without increasing the value of a_1 achieved. And this process of minimization continue till the last function a_ℓ has been minimized. The last result is the final solution of the problem. The principal aim of the GP approach is to attain the goals as closely as possible but always satisfying the higher priority goals before the lower level ones. Since the minimization of $\bar{a}$ takes place in the order of priority, the preemptive weights, P_i's, can be dropped from the final problem formulation. The complete GP model formulation is given below:

To find $X = (X_1, X_2, \ldots, X_n)^T$ so as to

$$\left.\begin{array}{ll} & \min \bar{a} = [a_1(d^-, d^+), a_2(d^-, d^+), \ldots, a_\ell(d^-, d^+)] \\ \text{subject to} & h_i(X) + d_i^- - d_i^+ = c_i, \; i = 1, \ldots, m \\ & f_i(X) + d_{m+i}^- - d_{m+i}^+ = b_i, \; i = 1, \ldots, k \\ & d^-, d^+ \geq 0, \; d_i^- \cdot d_i^+ = 0 \; \forall_i \end{array}\right| \quad (4)$$

Each achievement function, $a_j(d^-, d^+)$, is a linear function of the appropriate deviation variables. Each deviation variable is determined "independently" from the corresponding constraint equation as follows:

$$d_i^- = \begin{cases} d_i^- , & \text{if } d_i^- \geq 0 \\ 0 , & \text{if } d_i^- < 0 \end{cases}$$

$$\begin{aligned} &\text{where} \quad d_i^- = c_i - h_i\,(X) \\ &\text{or} \quad d_i^- = b_i - f_i\,(X) \end{aligned} \tag{5}$$

Similarly

$$d_i^+ = \begin{cases} d_i^+ , & \text{if } d_i^+ \geq 0 \\ 0 , & \text{if } d_i^+ \leq 0 \end{cases}$$

$$\begin{aligned} &\text{where} \quad d_i^+ = h_i(X) - c_i \\ &\text{or} \quad d_i = f_i(X) - b_i \end{aligned} \tag{6}$$

Notice that in the process of determining "each" deviation variable, the corresponding absolute or goal constraint, which is a function of the decision variables, $X = (X_1, X_2, \ldots, X_n)$, is utilized, so that the constraints in (4) are no longer as constraints to the minimization problem in the sense of constraints in single objective nonlinear programming problems.

By an iterative approach, the GP model can be decomposed into ℓ number of single objective problems as follows:

Problem 1: To find $X = (X_1, X_2, \ldots, X_n)$ so as to

$$\left.\begin{array}{l} \min \; a_1(d^-, d^+) \\ \text{subject to} \quad h_i(X) + d_i^- - d_i^+ = c_i, \quad i = 1,2, \ldots, m \\ d^-, d^+ \geq 0 \;\text{ and }\; d_i^- \cdot d_i^+ = 0 \;\forall_i \end{array}\right\} \quad (7)$$

Notice that the first priority level (Problem 1) is associated with the achievement of the absolute constraints. The last constraint, $d_i^- \cdot d_i^+ = 0$, implies that only one deviation variable, either positive or negative, exists in the solution.

Let a_1^* be the optimal solution for problem 1, i.e., $a_1^* = \min a_1(d^-, d^+)$. a_1^* is usually zero, since the absolute constraints must be satisfied. If so, there exists a solution for the GP problem. If $a_1^* \neq 0$, then the GP problem has no solution, i.e., the feasible region formed by the absolute objectives (constraints) is empty.

If $a_1^* = 0$, then the attainment problem for goal 1 is equivalent to problem 2.

Problem 2: To find X so as to

$$\min a_2 \,(d^-, d^+)$$

$$\text{subject to} \quad h_i(X) + d_i^- - d_i^+ = c_i, \quad i = 1,2, \ldots, m \tag{8}$$

$$a_1(d^-, d^+) \leq a_1^* \tag{9}$$

$$f_1(X) + d^-_{m+1} - d^+_{m+1} = b_1 \tag{10}$$

$$d^-, d^+ \geq 0 \quad \text{and} \quad d^+_i \cdot d^-_i = 0 \quad \forall_i$$

Notice that constraints (8) and (9) imply that in trying to achieve goal 1 we will not sacrifice our previously determined attainment of Problem 1.

Problem 3: To find X so as to

$$\min a_3 (d^-, d^+)$$

$$\text{subject to} \quad h_i(X) + d^-_i - d^+_i = c_i, \quad i = 1,2, \ldots, m$$

$$a_1(d^-, d^+) \leq a_1^*$$

$$f_1(X) + d^-_{m+1} - d^+_{m+1} = b_1$$

$$a_2(d^-, d^+) \leq a_2^*$$

$$f_2(X) + d^-_{m+2} - d^+_{m+2} = b_2$$

$$d^-, d^+ \geq 0 \quad \text{and} \quad d^+_i \cdot d^-_i = 0 \quad \forall_i$$

Let a_3^* be the solution for problem 3.

We can now write a general goal attainment problem (j+1) for attaining goal j, $0 \leq j \leq \ell-1$ as follow:

Problem (j+1): To find X so as to

$$\min a_{j+1}(d^-, d^+)$$

$$\text{subject to} \quad h_1(X) + d^-_i - d^+_i = c_i, \quad i = 1,2, \ldots, m$$

$$a_i(d^-, d^+) \le a_i^*, \quad i = 1,2, \ldots, j$$

$$f_i(X) + d_{m+i}^- - d_{m+i}^+ = b_i, \quad i = 1,2, \ldots, j$$

$$d^-, d^+ \ge 0 \text{ and } d_i^- . d_i^+ = 0 \quad \forall_i$$

2.1 Computational Procedure

The preceeding "ℓ" single objective decision making problems can be solved by any proper nonlinear programming method. The iterative approach to the nonlinear goal programming problem presented here follows closely to that of Dauer and Krueger [3]. The computational procedures which will be presented here for this iterative nonlinear goal programming method will be a modified Hooke and Jeeves pattern search. Using Hooke and Jeeves pattern search for nonlinear goal programming is presented by Ignizio [9]. We will present our computational procedures which integrate the iterative approach and the modified Hooke and Jeeves pattern search into an effective solution procedure.

One interesting feature of this procedure for solving the iterative nonlinear goal programming problem is that the problem is solved by traditional nonlinear search techniques that are originally intended for solving the so called "unconstrained" problem.

The original direct search method of Hooke and Jeeves [6,7] is a sequential search routine for minimizing an

"unconstrained" function $g(X)$ of more than one variable $X = (X_1, X_2, \ldots, X_n)$. The argument X is varied until the minimum of $g(X)$ is obtained. The search routine determines the sequence of values for X. The successive values of X can be interpreted as points in an n-dimensional space. The procedure consists of two types of moves: Exploratory and Pattern.

A move is defined as the procedure of going from a given point to the following point. A move is a success if the value of $g(X)$ decreases (for minimization); otherwise, it is a failure. The first type of move is an exploratory move which is designed to explore the local behavior of the objective function, $g(X)$. The success or failure of the exploratory moves is utilized by combining it into a pattern which indicates a probable direction for a successful move [6].

Since the GP problem is associated with constraints and deviation variables, the original Hooke and Jeeves pattern search method can not be applied directly for solving the problem. The method is modified. In the modified Hooke and Jeeves pattern search for NLGP, the procedure is to minimize an achievement function vector, $\bar{a} = (a_1, a_2, \ldots, a_\ell)$. In the iterative approach of the NLGP, solution to Problem (j+1) is to find $X = (X_1, X_2, \ldots, X_n)$ so as to minimize the achievement function, $a_{j+1}(d^-, d^+)$, such that the jth goal, $f_j(X) + d^-_{m+j} - d^-_{m+j} = b_j$, is satisfied and the previous attained

achievement functions are not violated, that is, $a_t(d^-, d^+) \leq a_t^*$, $t = 1,2, \ldots, j$. Therefore, the problem is a constrained problem. However, checking of the constraints, $a_t(d^-, d^+) \leq a_t^*$, $t = 1,2, \ldots, j$ can be integrated in a move of the modified Hooke and Jeeves pattern search.

A move is a <u>success</u> for the modified Hooke and Jeeves pattern search if the value of $a_{j+1}(d^-, d^+)$ decreases and $a_t(d^-, d^+) \leq a_t^*$, $t = 1,2, \ldots, j$ are satisfied; otherwise, it is a <u>failure</u>. The modifications are incorporated into the exploratory moves and pattern moves. The exploratory move is performed as follows:

1. Introduce a starting point X with a prescribed step length δ_i in each of the independent variables X_i, $i = 1,2, \ldots, n$.
2. Compute the achievement function, $a_{j+1}(d^-, d^+)$, where d^- and d^+ are functions of the decision variables, $X = (X_1, X_2, \ldots, X_n)$. Let $a_{j+1}(d^-, d^+) = a_{j+1}[X]$ Set $i = 1$.
3. Compute $a_{t+1}^i[X]$, $t = 1,2, \ldots, j$, at the trial point $X = (X_1, X_2, \ldots, X_i + \delta_i, \ldots, X_n)$.
4. Compare $a_{j+1}^i[X]$ with $a_{j+1}[X]$:

(i) If $a^i_{j+1}[X] < a_{j+1}[X]$, and $a^i_t[X] \leq a^*_t$ for $t = 1,2,\ldots,j$, set $a_{j+1}[X] = a^i_{j+1}[X]$, $X = (X_1, X_2, \ldots, X_n) =$

$$(X_1, X_2, \ldots, X_i + \delta_i, \ldots, X_n),$$

and $i = i + 1$. Consider this trial point as a starting point, and repeat from step 3.

(ii) If $a^i_{j+1}[X] \geq a_{j+1}[X]$ and/or $a^i_t[X] > a^*_t$ for any $t=1,2, \ldots, j$, set $X = (X_1, X_2, \ldots, X_i - 2\delta_i, \ldots, X_n)$. Compute $a^i_{t+1}[X]$, $t = 1,2, \ldots, j$, and see if $a^i_{j+1}[X] < a_{j+1}[X]$ and $a^i_t[X] \leq a^*_t$ for all $t = 1,2, \ldots, j$. If this move is a success the new trial point is retained. Set $a_{j+1}[X] = a^i_{j+1}[X]$, $X = (X_1, X_2, \ldots, X_i, \ldots, X_n) = (X_1, X_2, \ldots, X_i - 2\delta_i, \ldots, X_n)$ and $i = i+1$, and repeat from step 3. If again $a^i_{j+1}[X] \geq a_{j+1}[X]$ and/or $a^i_t[X] > a^*_t$ for any $t=1,2, \ldots, j$, then the move is a failure and X_i remains unchanged, that is, $X = (X_1, X_2, \ldots, X_i, \ldots, X_n)$ $= (X_1, X_2, \ldots, X_i + \delta_i, \ldots, X_n)$. Set $i = i+1$ and repeat from step 3.

The point X_B obtained at the end of the exploratory moves, which is reached by repeating step 3 until $i = n$ is defined as a base point. The starting point introduced in step 1 of the

exploratory move is a starting base point or point obtained by the pattern move.

The pattern move is designed to utilize the information acquired in the exploratory move, and executes the actual minimization of the function by moving in the direction of the established pattern. The pattern move is a simple step from the current base to the point.

$$X = X_B + (X_B - X_B^*)$$

X_B^* is either the starting base point or the preceding base point. Following the pattern move a series of exploratory moves is conducted to further improve the pattern. If the pattern move followed by the exploratory moves brings no improvement, the pattern move is a failure. Then we return to the last base which becomes a starting base and the process is repeated.

If the exploratory moves from any starting base do not yield a point which is better than this base, the lengths of all the steps are reduced and the moves are repeated. Convergence is assumed when the step lengths, δ_i, have been reduced below predetermined limits.

A descriptive flow diagram for the modified Hooke and Jeeves pattern search is given in Fig. 2.1.

After initializing a base point, the achievement functions $a_j[X]$, j = 1, 2, ..., are evaluated. In the process of evaluation,

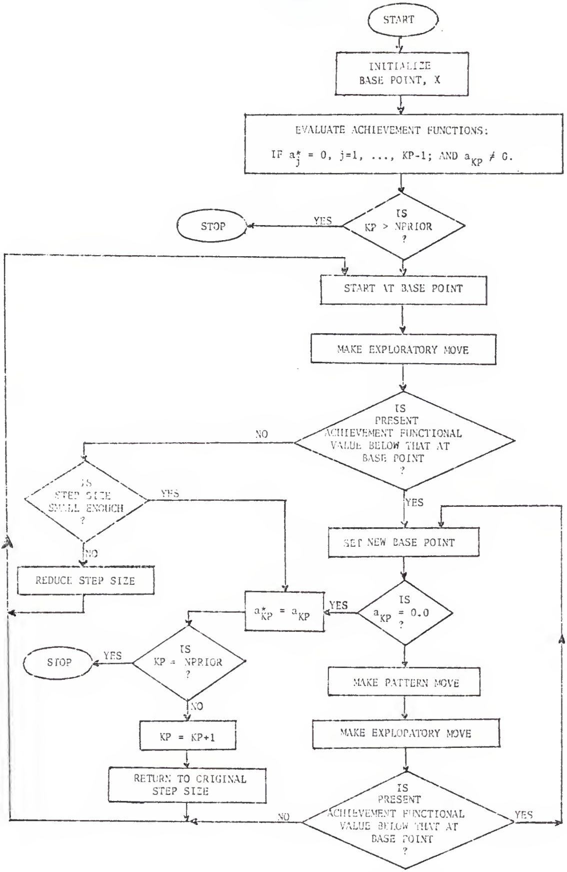

Fig.2.1. Flow diagram for the iterative NLGP algorithm with a modified Hooke and Jeeves pattern search.

we start with $j = 1$ and check if $a_1 = 0$. If so, a_2 is calculated and checked if $a_2 = 0$. This process will proceed until $j = KP$ when $a_{KP} \neq 0$. If KP is greater than the total number of the priorities, NPRIOR, then we get a solution. If KP is not greater than NPRIOR, then the modified Hooke and Jeeves pattern search is used for searching the solution for single objective decision problem, Problem KP. The procedures will be applied until KP = NPRIOR.

2.2 Numerical Example

The ABC company produces two similar products A and B. Both products are equally important. The total profit, in hundreds of dollars, can be approximated by the mathematical product of the two products in tons (X_1X_2), where X_1 and X_2 are dialy production of A and B in tons, respectively. The inprocess inventory costs of each product, in hundreds of dollars per ton, are $(X_1-4)^2$ and $(x_2)^2$ for products A and B, respectively. The labor cost of production is \$500/ton and \$400/ton for products A and B, respectively.

The president of the company has set the following goals in the order of their importance to the company.

(1) Limit the total cost of inprocess inventory to \$1025/day.
(2) Achieve the profit of at least \$300 per day, and limit the total labor cost to \$2,000 per day.

(3) The sum of one half of the daily production of product A and the daily production of product B should be more than 8 tons per day.

The problem may be formulated mathematically as follows:

Priority 1: The absolute objectives (constraints) are:

$$h_1(X) = X_1 \geq 0$$

$$h_2(X) = X_2 \geq 0$$

Priority 2: The first goal is to limit the inprocess inventory cost.

$$f_1(X) = (X_1-4)^2 + (X_2)^2 \leq 10.25$$

Priority 3: To achieve the profit and to limit the total labor cost are in the same priority level.

$$f_2(X) = X_1X_2 \geq 3$$

$$f_3(X) = 5X_1 + 4X_2 \leq 20$$

Priority 4: The last priority is to achieve the daily production goal.

$$f_4(X) = X_1 + 2X_2 \geq 8.$$

The NLGP problem in format of (4) will be:

To find X_1 and X_2 so as to

$$\min \bar{a} = [a_1, a_2, a_3, a_4] = [(d_1^- + d_2^-), (d_3^+), (d_4^- + d_5^+), (d_6^-)]$$

subject to $X_1 + d_1^- - d_1^+ = 0$

$$X_2 + d_2^- - d_2^+ = 0$$

$$(X_1-4)^2 + (X_2)^2 + d_3^- - d_3^+ = 10.25$$

$$X_1X_2 + d_4^- - d_4^+ = 3$$

$$5X_1 + 4X_2 + d_5^- - d_5^+ = 20$$

$$X_1 + 2X_2 + d_6^- - d_6^+ = 8$$

$$d^-, d^+ \geq 0, \quad d_i^- \cdot d_i^+ = 0 \quad \forall_i$$

By the iterative NLGP approach, the GP problem is decomposed into the following "4" single objective problems.

<u>Problem 1</u>: To find X_1 and X_2 so as to

$$\min \quad a_1 = d_1^- + d_2^-$$

subject to

$$X_1 + d_1^- - d_1^+ = 0$$

$$X_2 + d_2^- - d_2^+ = 0$$

$$d^-, d^+ \geq 0, \quad d_i^- \cdot d_i^+ = 0 \quad \forall_i$$

<u>Problem 2</u>: To find X_1 and X_2 so as to

$$\min \quad a_2 = d_3^+$$

subject to

$$X_1 + d_1^- - d_1^+ = 0$$

$$X_2 + d_2^- - d_2^+ = 0$$

$$a_1 \leq a_1^*$$

$$(X_1-4)^2 + (X_2)^2 + d_3^- - d_3^+ = 10.25$$

$$d^-, d^+ \geq 0, \quad d_i^- \cdot d_i^+ = 0 \quad \forall_i$$

Problem 3: To find X_1 and X_2 so as to

$$\min \quad a_3 = d_4^- + d_5^+$$

subject to

$$X_1 + d_1^- - d_1^+ = 0$$

$$X_2 + d_2^- - d_2^+ = 0$$

$$(X_1-4)^2 + (X_2)^2 + d_3^- - d_3^+ = 10.25$$

$$a_i \leq a_i^*, \quad i = 1, 2$$

$$X_1X_2 + d_4^- - d_4^+ = 3$$

$$5X_1 + 4X_2 + d_5^- - d_5^+ = 20$$

$$d^-, d^+ \geq 0, \quad d_i^- \cdot d_i^+ = 0 \qquad \forall_i$$

Problem 4: To find X_1 and X_2 so as to

$$\min \quad a_4 = d_6^-$$

subject to

$$X_1 + d_1^- - d_1^+ = 0$$

$$X_2 + d_2^- - d_2^+ = 0$$

$$(X_1-4)^2 + (X_2)^2 + d_3^- - d_3^+ = 10.25$$

$$X_1X_2 + d_4^- - d_4^+ = 3$$

$$5X_1 + 4X_2 + d_5^- - d_5^+ = 20$$

$$a_i \leq a_i^*, \quad i = 1, 2, 3$$

$$X_1 + 2X_2 + d_6^- - d_6^+ = 8$$

$$d^-, d^+ \geq 0, \quad d_i^- \cdot d_i^+ = 0 \qquad \forall_i$$

The NLGP problem is solved by the algorithm, presented in Fig. 2.1, which integrates the iterative approach and the modified Hooke and Jeeves pattern search.

Let a starting base point be $(X_1, X_2) = (8, 6)$. Then the achievement function of Problem 1, $a_1(d^-, d^+) = d_1^- + d_2^-$, is evaluated as follows (see (5) and (6)):

Since $d_1^- = c_1 - h_1 (X) = 0 - X_1 = -8 < 0$

set $d_1^- = 0$.

Similarlv

$$d_2^- = c_2 - h_2 (X) = 0 - X_2 = -6 < 0$$

set $d_2^- = 0$.

Therefore, $a_1(d_1^-, d_2^-) = d_1^- + d_2^- = 0$, which satisfies the absolute constraints. Let $a_1^* = \min a_1 = 0$. As shown in Fig. 2.2 a, any point in the first quadrant (the shaded area) will satisfy the absolute constraints and gives $a_1 = 0$.

The value of achievement function of Problem 2 at the base point, $a_2 = d_3^+$, is :

$$d_3^+ = f_1(X) - b_1 = (X_1 - 4)^2 + (X_2)^2 - 10.25 = 41.75 \neq 0$$

Therefore, KP = 2, and the modified Hooke and Jeeves pattern search is applied to find X so as to minimize

$$a_2 = d_3^+ = (X_1 - 4)^2 + (X_2)^2 - 10.25$$

To illustrate the modified procedure, the cost curve, $(X_1 - 4)^2+(X_2)^2= 10.25$, is drawn in Fig. 2.2 b. The numbers on the points indicate the sequence in which they are selected. The number on each point also corresponds to the number of functional values searched from the beginning of the problem 2 up to and including that point. Table 2.2 b presents the step by step results of applying the modified Hooke and Jeeves procedure for NLGP to problem 2.

The point $X^1 = (8, 6)$, is the starting base (B_{20}) which is also the last base point of problem 1. The step length is $= (\delta_1, \delta_2) = (0.5, 0.5)$. At the starting base point, X^1, exploratory moves are conducted first in X_1 direction. At the point, $X^2 = (8.5, 6.0)$, the values of achievement functions $a_2^1[X^2]$ and $a_1^1[X^2]$ are compared with $a_2[X^1] = 41.75$ and $a_1^* = 0$; $a_2^1[X^2] = 46 > a_2[X^1] = 41.75$ and $a_1^1[X^2] = 0 = a_1^*$. So the point X^2 is a failure. At $X^3 = (7.5, 6.0)$, $a_2^1[X^3] = 38 < a_2[X^1]=41.75$ and $a_1^1[X^3] = 0 = a_1^*$. So the point X^3 is a success, because both conditions are satisfied. Let $a_2[X] = a_2[X^3] = 38.0$. Again exploratory moves are conducted in X_2 direction at the point X^3. The point, $X^4 = (7.5, 6.5)$, is a failure because $a_2^2[X^4] = 44.25 > a_2[X^3] = 38$ although $a_1^2[X^4] = 0 = a_1^*$. The

point, $X^5 = (7.5, 5.5)$, is a success because $a_2^2[X^5] = 32.25 < a_2[X^3] = 38$ and $a_1^2[X^5] = 0 = a_1^*$. $X^5 = (7.5, 5.5)$ is the end of the exploratory moves and since X^5 is better point than X^1, X^5 is set as a new base point (B_{21}). Point $X^6 = (7,5)$ is obtained by the pattern move based on equation (11).

From $X^6 = (7, 5)$ exploratory moves are performed again; $X^{10} = (6.5, 4.5)$ becomes new base point because $a_2^2[X^{10}] = 16.25 < a_2[X^5] = 32.25$ and $a_1^2[X^{10}] = 0 = a_1^*$.

Point $X^{11} = (5.5, 3.5)$ is reached by the pattern move according to equation (11) where the last base point X_B^* is X^5 and the new base point is X^{10}.

Point, $X^{15} = (5.0, 3.0)$ is the result of the exploratory moves starting from point X^{11}, where moves to X^{13} and to X^{15} are successes because $a_2^1[X^{13}] < a_2[X^{11}]$ and $a_1^1[X^{13}] = a_1^*$ and $a_2^2[X^{15}] < a_2^1[X^{13}]$ and $a_2^2[X^{15}] = a_1^*$. Since $a_2^2[X^{15}] < a_2[X^{10}]$ and $a_1^2[X^{15}] = a_1^*$, X^{15} becomes a new base point.

At this point, $X^{15} = (5,3)$, because $a_2[X^{15}] = 0$ and $a_1[X^{15}] = 0$, it is a solution to problem 2.

Let $a_2^* = a_2[X^{15}] = 0$ (minimum of a_2). As shown in

Fig. 2.2 b, any point in the shaded area (II) will satisfy priority levels 1 and 2 completely. Now we will set KP = 2 + 1 = 3, and the problem 3, min $a_3 = d_4^- + d_5^+$, is solved by starting at the last base point obtained in problem 2. i.e., letting starting base point (B_{30}), $X^1 = (5, 3)$.

The step by step results of problem 3 are presented in table 2.1 c and Fig. 2.2 c. After series of pattern and exploratory moves, point $X^{13} = (3.0, 1.0)$ is obtained where priorities 1, 2, and 3 are completely satisfied because at this point, $a_1 = 0$, $a_2 = 0$ and $a_3 = 0$. Let $a_3^* = \min a_3 = 0$. As shown in Fig. 2.2 c, any point in the shaded area (III) satisfies the priority levels 1, 2, and 3.

The search procedure is again continued for solving the problem 4, min $a_4 = d_6^-$; after setting KP = 3 + 1 = 4 and starting base point, B_{40}, as the last base point obtained in problem 3, i.e., $B_{40} = (3.0, 1.0)$.

The step by step results of problem 4 are presented in table 2.1 d and in Fig. 2.2 d. Minimum value for a_4 is obtained at the point $X^{27} = (2.0, 2.5)$ where $a_1 = 0$, $a_2 = 0$, $a_3 = 0$ and $a_4 = 1.0$. All attempts to reduce the value of a_4 from 1.0 have failed because a_1^*, a_2^*, and a_3^* are getting increased at any other point where a_4 is less than 1.0,

which is highly undesirable. The shaded area (III) in Fig. 2.2 d represents the feasible region for priorities 1, 2, and 3; shaded area (IV) represents the feasible region for priority level 4. It is evident that we can not attain priority level 4 (goal 3) completely because there is no common region formed by the feasible regions (III) and (IV). So the optimal solution for the GP problem is $X_1 = 2$, and $X_2 = 2.5$. All absolute constraints are satisfied and goals 1 and 2 are completely achieved, but goal 3 is not achieved fully.

Table 2.1a. Step by step results of Problem 1 of the iterative NLGP problem.

n	Base point X_B	Step size	Better point X	a_1	Trial point X^n	a_1	Comments
1	B_{10}	(.5,.5)	(8.0, 6.0)	0.0			Priority 1 satisfied.

Table 2.1b. Step by step results of Problem 2 of the iterative NLGP problem.

n	Base point X_B	Step size	Better point X	a_j^*: a_1^*	a_j^*: a_2	Trial point X^n	a_j: a_1	a_j: a_2	Comments
1	$B_{20} = B_{10}$	(.5, .5)	(8.0, 6.0)	0.0	41.75				Starting Base Pt.
2			(8.0, 6.0)	0.0	41.75	(8.5, 6.0)	0.0	46.0	Exp. Fail.
3			(8.0, 6.0)	0.0	41.75	(7.5, 6.0)	0.0	38.0	Exp. Succ.
4			(7.5, 6.0)	0.0	38.0	(7.5, 6.5)	0.0	44.25	Exp. Fail.
5			(7.5, 6.0)	0.0	38.0	(7.5, 5.5)	0.0	32.25	Exp. Succ.
5	B_{21}		(7.5, 5.5)	0.0	32.25				
6			(7.5, 5.5)	0.0	32.25	(7.0, 5.0)	0.0	23.75	Pattern Move Succ.
7			(7.0, 5.0)	0.0	23.75	(7.5, 5.0)	0.0	27.0	Exp. Fail.
8			(7.0, 5.0)	0.0	23.75	(6.5, 5.0)	0.0	21.0	Exp. Succ.
9			(6.5, 5.0)	0.0	21.0	(6.5, 5.5)	0.0	26.25	Exp. Fail.
10			(6.5, 5.0)	0.0	21.0	(6.5, 4.5)	0.0	16.25	Exp. Succ.
10	B_{22}	(.5, .5)	(6.5, 4.5)	0.0	16.25				
11			(6.5, 4.5)	0.0	16.25	(5.5, 3.5)	0.0	4.25	Pattern (Success)
12			(5.5, 3.5)	0.0	4.25	(6.0, 3.5)	0.0	6.0	Exp. Fail.
13			(5.5, 3.5)	0.0	4.25	(5.0, 3.5)	0.0	3.0	Exp. Succ.
14			(5.0, 3.5)	0.0	3.0	(5.0, 4.0)	0.0	6.75	Exp. Fail.
15			(5.0, 3.5)	0.0	3.0	(5.0, 3.0)	0.0	0.0	Exp. Succ.
15	B_{23}	(.5, .5)	(5.0, 3.0)	0.0	0.0				Priorities 1 & 2 satisfied.

Table 2.1c. Step by step results of Problem 3 of the iterative NLGP problem.

n	Base point X^n	Step size	Better point X	a_1^*	a_2^*	a_3	Trial point X^n	a_1	a_2	a_3	Comments
1	$B_{30} = B_{23}$	(.5,.5)	(5.0, 3.0)	0.0	0.0	17.0					Starting Base Pt.
2			(5.0, 3.0)	0.0	0.0	17.0	(5.5, 3.0)	0.0	1.0	19.5	Fail
3			(5.0, 3.0)	0.0	0.0	17.0	(4.5, 3.0)	0.0	0.0	14.5	Succ.
4			(4.5, 3.0)	0.0	0.0	14.5	(4.5, 3.5)	0.0	2.25	16.5	Fail.
5			(4.5, 3.0)	0.0	0.0	14.5	(4.5, 2.5)	0.0	0.0	12.5	Succ.
5	B_{31}		(4.5, 2.5)	0.0	0.0	12.5					
6			(4.5, 2.5)	0.0	0.0	12.5	(4.0, 2.0)	0.0	0.0	8.0	Pattern Succ.
7			(4.0, 2.0)	0.0	0.0	8.0	(4.5, 2.0)	0.0	0.0	10.5	Fail.
8			(4.0, 2.0)	0.0	0.0	8.0	(3.5, 2.0)	0.0	0.0	5.5	Succ.
9			(3.5, 2.0)	0.0	0.0	5.5	(3.5, 2.5)	0.0	0.0	7.5	Fail.
10			(3.5, 2.0)	0.0	0.0	5.5	(3.5, 1.5)	0.0	0.0	3.5	Succ.
10	B_{32}		(3.5, 1.5)	0.0	0.0	3.5					
11			(3.5, 1.5)	0.0	0.0	3.5	(2.5, 0.5)	0.0	0.0	1.75	Pattern Succ.
12			(2.5, 0.5)	0.0	0.0	1.75	(3.0, 0.5)	0.0	0.0	1.5	Succ.
13			(3.0, 0.5)	0.0	0.0	1.5	(3.0, 1.0)	0.0	0.0	0.0	Succ.
13	B_{33}		(3.0, 1.0)	0.0	0.0	0.0					
											Priorities 1,2 & 3 satisfied.

Table 2.1d. Step by step results of Problem 4 of the iterative NLCP problem.

n	Base point X_B	Step size	Better point X	a_1^*	a_2^*	a_3^*	a_4	Trial point X^n	a_1	a_2	a_3	a_4	Comments
1	$B_{40} = B_{33}$	(.5,.5)	(3.0, 1.0)	0.0	0.0	0.0	3.0						Starting Base Pt.
2			(3.0, 1.0)	0.0	0.0	0.0	3.0	(3.5, 1.0)	0.0	0.0	1.5	2.5	Fail.
3			(3.0, 1.0)	0.0	0.0	0.0	3.0	(2.5, 1.0)	0.0	0.0	0.5	3.5	Fail.
4			(3.0, 1.0)	0.0	0.0	0.0	3.0	(3.0, 1.5)	0.0	0.0	1.0	2.0	Fail.
5			(3.0, 1.0)	0.0	0.0	0.0	3.0	(3.0, 0.5)	0.0	0.0	1.5	4.0	Fail.
1	B_{40}	(.25,.25)	(3.0, 1.0)	0.0	0.0	0.0	3.0						Starting Base
6			(3.0, 1.0)	0.0	0.0	0.0	3.0	(3.25,1.0)	0.0	0.0	0.25	2.75	Fail.
7			(3.0, 1.0)	0.0	0.0	0.0	3.0	(2.75,1.0)	0.0	0.0	0.25	3.25	Fail.
8			(3.0, 1.0)	0.0	0.0	0.0	3.0	(3.0,1.25)	0.0	0.0	0.0	2.5	Succ.
8	B_{41}	(.25,.25)	(3.0,1.25)	0.0	0.0	0.0	2.5						
9			(3.0,1.25)	0.0	0.0	0.0	2.5	(3.0, 1.5)	0.0	0.0	1.0	2.0	Pattern Move Fail.
10			(3.0,1.25)	0.0	0.0	0.0	2.5	(3.25,1.5)	0.0	0.0	2.25	1.75	Fail.
11			(3.0,1.25)	0.0	0.0	0.0	2.5	(2.75,1.5)	0.0	0.0	0.0	2.25	Succ.
12			(2.75,1.5)	0.0	0.0	0.0	2.25	(2.75,1.75)	0.0	0.0	0.75	1.75	Fail.
13			(2.75,1.5)	0.0	0.0	0.0	2.25	(2.75,1.25)	0.0	0.0	0.0	2.75	Fail.

Table 2.1d (cont'd).

n	Base point X_B	Step size	Better point X	a_1^*	a_2^*	a_3^*	a_4	Trial point X^n	a_1	a_2	a_3	a_4	Comments
11	B_{42}		(2.75, 1.5)	0.0	0.0	0.0	2.25						
14			(2.75, 1.5)	0.0	0.0	0.0	2.25	(2.5, 1.75)	0.0	0.0	0.0	2.0	Pattern (Success)
15			(2.5, 1.75)	0.0	0.0	0.0	2.0	(2.75,1.75)	0.0	0.0	0.75	1.75	Fail.
16			(2.5, 1.75)	0.0	0.0	0.0	2.0	(2.75,1.75)	0.0	0.0	0.0	2.25	Fail.
17			(2.5, 1.75)	0.0	0.0	0.0	2.0	(2.5, 2.0)	0.0	0.0	0.0	2.0	Fail.
18			(2.5, 1.75)	0.0	0.0	0.0	2.0	(2.5, 1.5)	0.0	0.0	0.0	2.5	Fail.
14	B_{43}	(.25,.25)	(2.5, 1.75)	0.0	0.0	0.0	2.0						
19			(2.5,.1.75)	0.0	0.0	0.0	2.0	(2.25, 2.0)	0.0	0.0	0.0	1.75	Pattern (Success)
20			(2.25, 2.0)	0.0	0.0	0.0	1.75	(2.5, 2.0)	0.0	0.0	0.5	1.5	Fail.
21			(2.25, 2.0)	0.0	0.0	0.0	1.75	(2.0, 2.0)	0.0	0.0	0.0	2.0	Fail.
22			(2.25, 2.0)	0.0	0.0	0.0	1.75	(2.25,2.25)	0.0	0.0	0.25	1.25	Fail.
23			(2.25, 2.0)	0.0	0.0	0.0	1.75	(2.25,1.75)	0.0	0.0	0.0	2.25	Fail.
19	B_{44}		(2.25, 2.0)	0.0	0.0	0.0	1.75						
24			(2.25, 2.0)	0.0	0.0	0.0	1.75	(2.0, 2.25)	0.0	0.0	0.0	1.5	Pattern (Success)
25			(2.0, 2.25)	0.0	0.0	0.0	1.5	(2.25,2.25)	0.0	0.0	0.25	1.25	Fail.

Table 2.1d (cont'd).

n	Base point X_B	Step size	Better point X	a_j^*				Trial point X^n	a_j				Comments
				a_1^*	a_2^*	a_3^*	a_4		a_1	a_2	a_3	a_4	
26			(2.0, 2.25)	0.0	0.0	0.0	1.5	(1.75, 2.25)	0.0	0.0	0.0	1.75	Fail.
27			(2.0, 2.25)	0.0	0.0	0.0	1.5	(2.0., 2.5)	0.0	0.0	0.0	1.0	Succ.
27	B_{45}		(2.0, 2.50)	0.0	0.0	0.0	1.0						
28			(2.0, 2.5)	0.0	0.0	0.0	1.0	(1.75, 3.0)	0.0	3.81	0.75	0.25	Pattern Fail
29			(2.0, 2.5)	0.0	0.0	0.0	1.0	(2.0 , 3.0)	0.0	2.75	2.0	0.0	Fail.
30			(2.0, 2.5)	0.0	0.0	0.0	1.0	(1.5 , 3.0)	0.0	5.0	0.0	0.5	Fail.
31			(2.0, 2.5)	0.0	0.0	0.0	1.0	(1.75,3.25)	0.0	5.38	1.75	0.0	Fail.
32			(2.0, 2.5)	0.0	0.0	0.0	1.0	(1.75,2.75)	0.0	2.38	0.0	0.75	Fail.
27	B_{45}	(.125, .125)	(2.0, 2.5)	0.0	0.0	0.0	1.0						Starting Base
33			(2.0, 2.5)	0.0	0.0	0.0	1.0	(2.125, 2.5)	0.0	0.0	0.63	0.88	Fail.
34			(2.0, 2.5)	0.0	0.0	0.0	1.0	(1.875, 2.5)	0.0	0.52	0.0	1.13	Fail.
35			(2.0, 2.5)	0.0	0.0	0.0	1.0	(2.0, 2.625)	0.0	0.64	0.5	0.75	Fail.
36			(2.0, 2.5)	0.0	0.0	0.0	1.0	(2.0, 2.375)	0.0	0.0	0.0	1.25	Fail.

Table 2.1d (cont'd).

n	Base point X_B	Step size	Better point X	a_1^*	a_2^*	a_3^*	a_4^*	Trial point X''	a_1	a_2	a_3	a_4	Comments
27	B_{45}	(.0625, .0625)	(2.0, 2.5)	0.0	0.0	0.0	1.0						Starting base
37			(2.0, 2.5)	0.0	0.0	0.0	1.0	(2.0625, 2.5)	0.0	0.0	0.31	0.94	Fail.
38			(2.0, 2.5)	0.0	0.0	0.0	1.0	(1.9375, 2.5)	0.0	0.25	0.0	1.06	Fail.
.			.	.	.	.	.	.	.	.	.	.	
.			.	.	.	.	.	.	.	.	.	.	
.			.	.	.	.	.	.	.	.	.	.	
	B_{45}		(2.0, 2.5)	0.0	0.0	0.0	1.0	.					Optimal solution

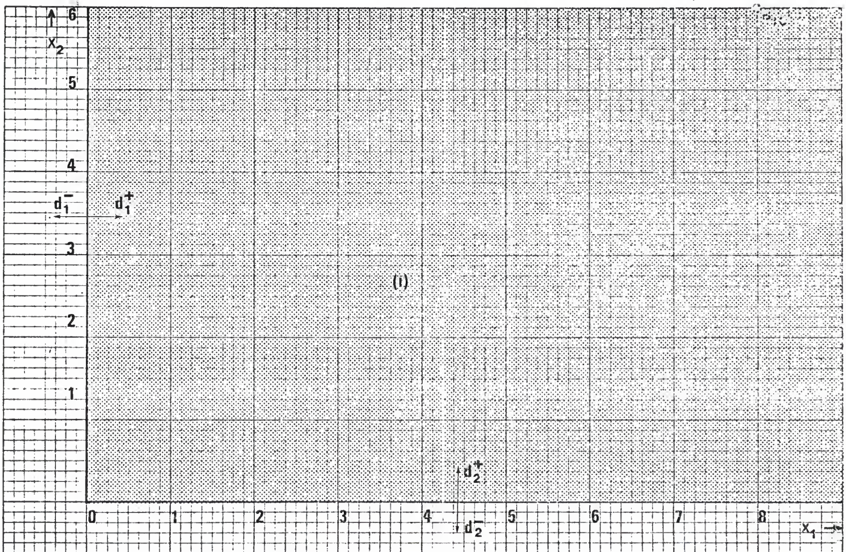

Fig. 2.2a. Solution to Problem 1 of the NLGP problem.

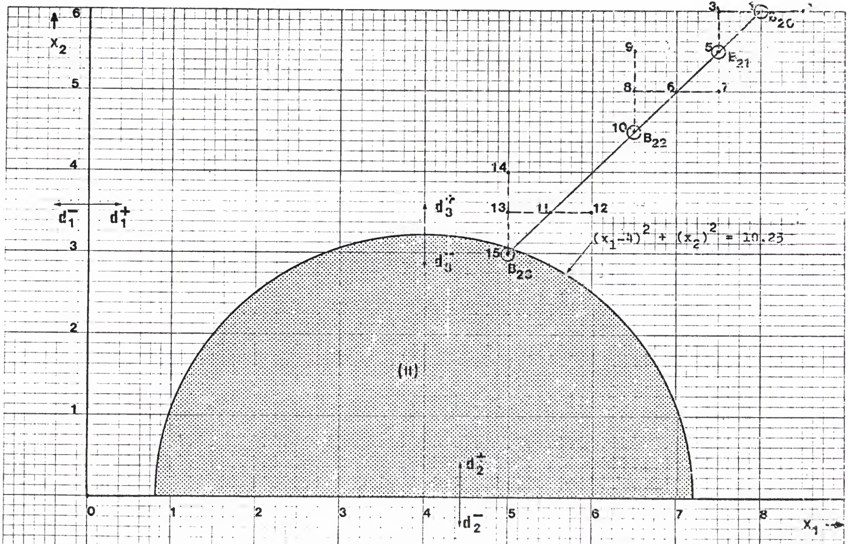

Fig. 2.2b. Solution to Problem 2 of the NLGP problem.

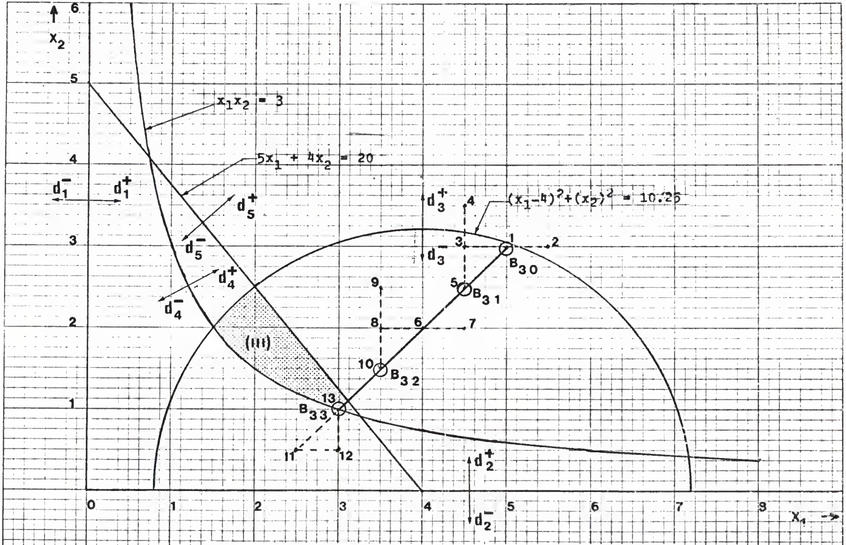

Fig. 2.2c. Solution to Problem 3 of the NLGP problem.

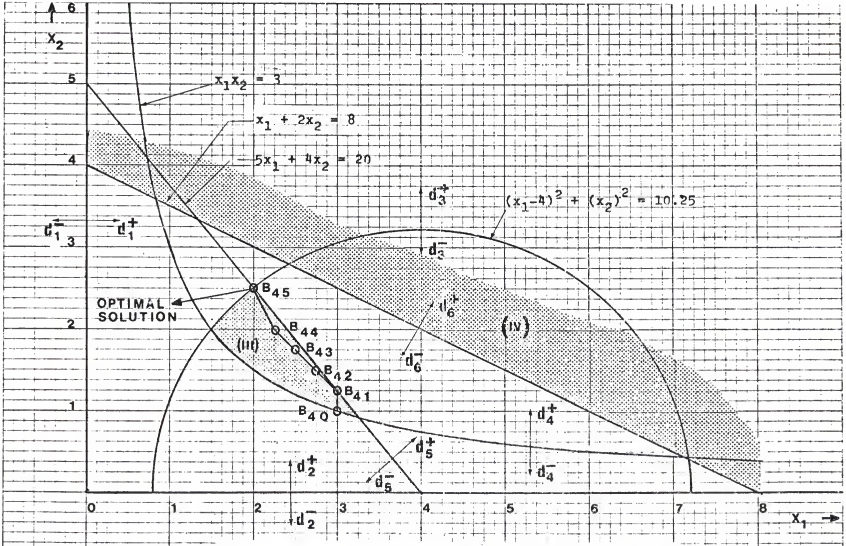

Fig. 2.2d. Solution to Problem 4 of the NLGP problem.

CHAPTER 3

APPLICATION OF NONLINEAR GOAL PROGRAMMING TO PRODUCTION PLANNING

3.1 Introduction

For any firm, it is important to achieve the most efficient utilization of available resources while meeting the restrictions imposed by the environment and by organizational policies concerning employment, inventories, production and subcontracting.

The most difficult problem encountered in aggregate production planning is when the problem is dynamic, i.e., when the demand rate varies over time. The fluctuations in demand can be absorbed by adopting one of or a combination of the following strategies.

1. Adjusting the capacity by changing size of the work force through hiring or laying off employees.

2. Using overtime in peak periods or idle time in slack periods to vary output while maintaining constant work force.

3. Use of subcontracting in peak periods.

4. Adjusting the inventory level to absorb fluctuations in demand.

In actual work settings, however, aggregate production

planning is further complicated by other factors such as variability of material costs according to the size, employee's willingness to work overtime, accuracy in sales forecasts, accuracy in the estimation of cost coefficients. The aggregate production planning strategy, which is shown in Fig. 3.1, is a dynamic process that relates demand and shipment of goods.

Many methods for finding the optimal strategy have been suggested, but none of these suggested methods has found any widespread use in industry. One of the reasons seems to be that the proposed models are gross oversimplifications of reality, and moreover, they do not provide room to reflect management's preferences or policies in the solution. Therefore, an effective application of such methods may be possible only at the expense of changing organizational policies.

The difficulty with the single objective model is not so much in its inability to represent the complexities of reality. Rather, the difficulty lies in the fact that its application requires cost information that is often very hard to estimate, for example, cost of hiring and layoff work force, correct costs of carrying inventory, opportunity costs of tying up capital in inventory, the actual costs of stockouts. Goal programming technique can handle these problems by considering with these costs as decision making problem. There is no stipulation that all the units of the objectives should be commensurable in the goal programming approach.

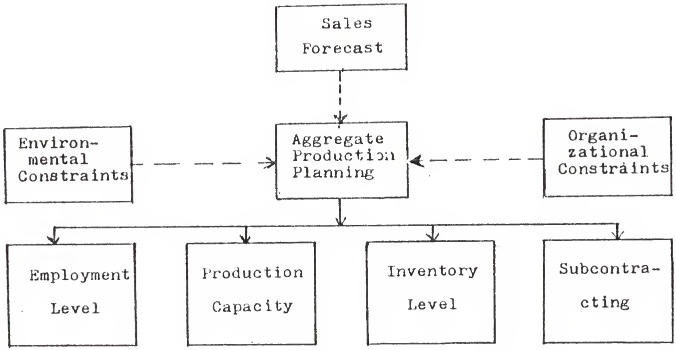

Fig. 3.1 . Aggregate production planning strategy.

3.2 Application of Nonlinear Goal Programming for Production Planning

Lee [12] applied linear goal programming technique for solving aggregate production planning problems involving linear multiple objectives. In practical applications of aggregate production planning problems it may often be the situation that one or more objectives are found to be nonlinear in nature. Goodman [4] reformulated the well known classic model of Holt et al. [5] into a goal programming model by linear approximation of the original objective function. However, it may not be possible to linearize all the nonlinear objective functions, even if it is done it may lead to sub-optimal solutions.

In this chapter the nonlinear goal programming approach as discussed in chapter II is applied for solving the non-linear aggregate production planning problem.

3.3 Numerical Example

The well known classical model of Holt et al. [5] is modified. Two more objectives to the original objective of minimizing the cost are added. The schematic representation of the problem is shown in Fig. 3.2.

Let

n = a month in the planning horigon

N = the duration, in months

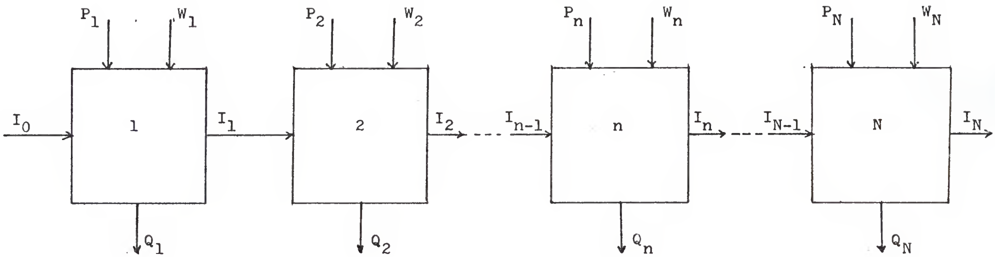

Fig. 3.2. Block diagram for personnel and production scheduling.

W_n = work force level in the nth month (workers)

P_n = Production rate at the nth month (units/month).

Q_n = Sales rate at the nth month (units/month).

I_n = Inventory level at the end of the nth month (units).

Inventory level at the end of each month is computed by using the recursive relationship between sales, production and inventory as follows:

$$I_n = I_{n-1} + P_n - Q_n , \quad n = 1,2, \ldots, N$$

The Holts model [5] to the paint factory problem considers that the total operating costs consists of the following cost terms:

1. <u>Regular payroll cost</u>

Regular payroll cost = $C_1W_n + C_2$ (\$/month)

where C_1 is the payroll cost (\$/man.month)

C_2 is the fixed cost (\$/month)

2. <u>Hiring and layoff costs</u>

The hiring cost is normally proportional to the number of workers hired. But certain random factors may affect the cost of hiring; e.g., how much difficulty is experienced in a particular case of hiring a man of desired qualifications and similarly the efficiency of hiring, measured in terms of quality of the employees hired, may fall when a large number

of people are hired at one time. So, the hiring cost can be approximated by a quadratic approximation can give a tolerable approximation over a range. Hence,

$$\text{Hiring cost} = C_3(W_n - W_{n-1})^2 + C_{14} \text{ (\$/month)},$$

$$W_n - W_{n-1} = \begin{cases} = 0, \text{ if } W_n \leq W_{n-1} \\ = W_n - W_{n-1}, \text{ if } W_n > W_{n-1} \end{cases}$$

where C_3 is a constant (\$/man^2.month)

C_{14} is a constant (\$/month)

3. Layoff cost

The layoff cost is normally proportional to the number of workers laid off. But certain random factors may affect the cost of layoff; e.g., how much reorganization is required in making a particular reduction in work force. The layoff cost can be approximated by a quadratic equation over a range as follows:

$$\text{Layoff cost} = C_4(W_n - W_{n-1})^2 + C_{15} \text{ (\$/month)},$$

$$W_n - W_{n-1} = \begin{cases} = 0, \text{ if } W_n \geq W_{n-1} \\ = W_n - W_{n-1}, \text{ if } W_n < W_{n-1} \end{cases}$$

where C_4 is a constant (\$/man^2.month)

C_{15} is a constant (\$/month)

We can obtain a single equation for both hiring and layoff costs, if we assume that $C_3 = C_4$ and $C_{14} = C_{15}$. So,

hiring and layoff cost = $C_3(W_n - W_{n-1})^2 + C_{14}$ (\$/month)

4. Overtime cost

The overtime cost is dependant upon two decision variables, the size of the workforce, W_n, and the production rate, P_n. With a given workforce, W_n, and an average worker productivity, C_5(units/man.month), the expression C_5W_n is the maximum number of units that can be produced in a month without incurring any overtime. In order to produce at higher rates than C_5W_n, overtime is required, and its amount increases with increased production. So,

Overtime cost = $C_6(P_n - C_5W_n)$ (\$/month)

where C_6 is the overtime cost per unit.

The above relation holds good only if there are no random disturbances in the production process. The estimated overtime costs must depend on an estimate of the probabilities that such disturbances will occur. The quadratic curve that approximates the expected cost of overtime for a given size, W_n, of workforce, and for different production rates is:

Expected cost of overtime = $C_7(P_n - C_5W_n)^2 + C_8P_n - C_9W_n + C_{10}P_nW_n$ (\$/month)

where C_7 is a constant ($.month/unit^2)

C_8 is a constant ($/unit)

C_9 is a constant ($/man.month)

C_{10} is a constant ($/man.unit)

In the above expected overtime cost equation if production falls to a very low level relative to the workforce, the overtime cost predicted by the quadratic curve rises and the approximation to the original cost curve becomes poor. However, the quadratic may be quite adequate approximation in the relevent range.

5. Inventory and back order costs

From lot size formulas it is known that both the optimal batch size and the optimal safety stock increase roughly as the square root of the order rate, Q_n. Thus the optimal aggregate inventory must increase with increased aggregate order rate, Q_n. The total expected back orders corresponding to any given size of inventory also must increase with an increased order rate. By combining these two relationships it appears that optimal net inventory increases with the order rate. The relationship between optimal net inventory and aggregate order rate may be approximated over a limited range by a function of the form:

optimal net inventory at the end of month $= C_{11} + C_{12}Q_n$ (units)

where C_{11} is a constant (units)

C_{12} is a constant (months)

When actual net inventory deviates from the optimal net inventory $(C_{11} + C_{12}Q_n)$, in either direction, costs rise. If net inventory falls below this optimal level, then the safety stock and batch sizes must be reduced. The rise in costs as net inventory declines can be estimated by costing the increased number of machine setups, the increased back orders and decreased inventory. Similarly, costs of inventory can be calculated when the net inventory is above the optimal level. Over a range, the curves of inventory-related costs may be approximated by a quadratic equation as follows:

$$\text{Expected inventory costs} = C_{13}\,[\,I_n - (C_{11} + C_{12}Q_n)\,]^2 ,$$

$$I_n - (C_{11} + C_{12}Q_n) = \begin{cases} = 0, & \text{if } I_n \leqslant C_{11} + C_{12}Q_n \\ = I_n - (C_{11} + C_{12}Q_n), & \text{if } I_n > C_{11} + C_{12}Q_n \end{cases}$$

Similarly,

$$\text{Expected back order costs} = C_{16}\,[\,I_n - (C_{11} + C_{12}Q_n)\,]^2 ,$$

$$I_n - (C_{11} + C_{12}Q_n) = \begin{cases} = 0, & \text{if } I_n \geqslant C_{11} + C_{12}Q_n \\ = I_n - (C_{11} + C_{12}Q_n), & \text{if } I_n < C_{11} + C_{12}Q_n \end{cases}$$

where C_{13} is a constant ($\$/\text{month.unit}^2$)

C_{14} is a constant ($/month.unit^2)

If we assume that $C_{13} = C_{16}$, then

expected inventory and back order cost = $C_{13} [I_n - (C_{11} + C_{12}Q_n)]^2$

Now we can obtain the total cost equation by adding the relevent costs

$$\begin{aligned} \text{Total cost} &= (\text{payroll cost}) + (\text{Hiring and layoff costs}) + \\ &\quad (\text{overtime cost}) + (\text{Inventory and back order costs}) \\ &= [C_1 W_n + C_2] + [C_3 (W_n - W_{n-1})^2 + C_{14}] + \\ &\quad [C_7 (P_n - C_5 W_n)^2 + C_8 P_n - C_9 W_n + C_{10} P_n W_n] + \\ &\quad \{C_{13} [I_n - (C_{11} + C_{12} Q_n)]^2\} \end{aligned}$$

In the total cost equation, the constant cost terms, C_2 and C_{14} can be dropped because they will not affect the decision in selecting the optimal decision variables, P_n and W_n.

The above model was set up by Holt et.al [5] and applied to a paint factory problem. He evaluated the constants after applying the model to the actual data.

$C_1 = 340$ ($/man.month)	$C_9 = 281$ ($/unit)
$C_3 = 64.3$ ($/man^2.month)	$C_{10} = 0$
$C_5 = 5.67$ (units/man.month)	$C_{11} = 320$ (units)
$C_7 = 0.20$ ($.month/unit^2)	$C_{12} = 0$
$C_8 = 51.2$ ($/unit)	$C_{13} = 0.0825$

So, the total cost equation becomes

$$\text{Total cost} = [340\ W_n] + [64.3\ (W_n - W_{n-1})^2] +$$
$$[0.2\ (P_n - 5.67W_n)^2 + 51.2P_n - 281W_n] +$$
$$[0.0825\ (I_n - 320.0)^2] \qquad (\$/\text{month})$$

In the total cost equation, the quadratic equation for hiring and layoff is fitted to the actual data of the paint factory problem in the range $-15 \leq W_n - W_{n-1} \leq 15$. The equation for the Inventory and back log cost was fitted in the range $-600 \leq I_n \leq 600$.

The system then can be represented by the following single objective model.

$$\text{Min } Z = \sum_{n=1}^{N} \Big[(340.0W_n) + 64.3\ (W_n - W_{n-1})^2 + 0.2\ (P_n - 5.67W_n)^2 + 51.2P_n - 281.0W_n + 0.0825\ (I_n - 320)^2 \Big]$$

subject to

$$I_n = I_{n-1} + P_n - Q_n \geq 0\ , \quad n = 1, 2, \ldots, N$$

and $0.2(P_n - 5.67W_n)^2 + 51.2P_n - 281.0W_n \geq 0\ , \quad n = 1, 2, \ldots, N$

The reason for considering the non-negative overtime cost is due to the characteristics of its mathematical formula. Taubert[14] found that minimizing the total cost over the planning period by selecting a certain W_n and P_n combination contributed negative overtime cost. Since the negative overtime cost is illogical in the context of the original paint factory example, the constraint of the non-negative cost should be imposed. The above problem can be solved by using any proper single objective optimization technique.

However, if the manager of the paint factory has some additional goals to be achieved apart from minimizing the cost, the problem becomes a multiobjective model. Let the manager has the following goals to be achieved.

(1) Limit the average stockouts to 1%

(2) Limit the total cost to $127,000

(3) Limit the average employees laid off to 1%

Two models are considered here by interchanging the priority of goals (2) and (3). The two models are formulated into a goal programming model as shown below.

<u>Model 1</u>:

(1) <u>Absolute constraints</u>:

$$0.2\ (P_n - 5.67W_n)^2 + 51.2\ P_n - 281.0\ W_n + d_n^- - d_n^+ = 0$$

$$n = 1, 2, \ldots, N$$

where d_n^- and d_n^+ are negative overtime and positive overtime cost.

(2) Goal constraints:

(a) Average stockouts:

$$\frac{100}{\sum_{n=1}^{N} Q_n} \left[\sum_{n=1}^{N} (-I_n) \right] + d_{N+1}^- - d_{N+1}^+ = 1,$$

$$I_n = \begin{cases} = 0 \ , \text{ if } I_n \geq 0 \\ = I_n \ \text{ if } I_n < 0 \end{cases}$$

where d_{N+1}^- and d_{N+1}^+ represent underachievement and over-achievement of stockout goal, respectively.

(b) Total cost:

$$C = \sum_{n=1}^{N} \Big[(340.0W_n) + 64.3\,(W_n - W_{n-1})^2$$
$$+ [\, 0.20(P_n - 5.67W_n)^2 + 51.2\, P_n - 281.0\, W_n]$$
$$+ 0.0825\,(I_n - 320.0)^2 \Big] + d_{N+2}^- - d_{N+2}^+ = 127{,}000$$

where d_{N+2}^- and d_{N+2}^+ represent under utilization and over utilization of budget, respectively.

(c) Percentage of average employees laid off:

$$\frac{100}{\sum_{n=1}^{N} W_n} \left[\sum_{n=1}^{N} (W_{n-1} - W_n) \right] + d_{N+3}^- - d_{N+3}^+ = 1,$$

$$W_{n-1} - W_n = \begin{cases} = W_{n-1} - W_n, \text{ if } W_{n-1} > W_n \\ = 0 \ , \text{ if } W_{n-1} \leq W_n \end{cases}$$

where d_{N+3}^- and d_{N+3}^+ represent under achievement and over achievement of goal 3 respectively.

(3) The production balance constraints are

$$I_n = I_{n-1} + P_n - Q_n \ , \ n = 1,2, \ \ldots, \ N$$

In addition to the variables and constraints above, the following preemptive priority factors are defined in order to pursue the various stated goals.

P_1 : The highest priority is assigned to minimization of the negative overtime cost (d_n^-, n = 1,2, ..., N)

P_2 : The second priority is assigned to minimization of the over utilization of the allowed percentage of average stock-outs.

P_3 : The third priority is assigned to minimizing the over utilization of allowed budget.

P_4 : The fourth priority is assigned to minimizing the over utilization of the percentage of employees laid off.

The complete GP model is:

Choose the optimal values for production rate, P_n, and work force level, W_n, at each month of the planning origin so as to

$$\text{Min a} = [P_1 \ (\sum_{n=1}^{N} d_n^-), \ P_2 \ (d_{N+1}^+), \ P_3 \ (d_{N+2}^+), + P_4 \ (d_{N+3}^+)]$$

subject to $\quad 0.20(P_n - 5.67W_n)^2 + 51.2 \ P_n - 281.0 \ W_n +$

$$d_n^- - d_n^+ = 0 \ , \ n = 1,2, \ \ldots, \ N$$

$$\frac{100}{\sum_{n=1}^{N} Q_n} \times \sum_{n=1}^{N} (-I_n) + d_{N+1}^{-} - d_{N+1}^{+} = 1, \quad I_n = \begin{cases} = 0, & \text{if } I_n \geq 0 \\ = I_n, & \text{if } I_n < 0 \end{cases}$$

$$\sum_{n=1}^{N} [340\ W_n] + [64.3\ (W_n - W_{n-1})^2] + [0.20\ (P_n - 5.67\ W_n)^2 + 51.2\ P_n - 281.0\ W_n] + [0.0825\ (I_n - 320)^2] + d_{N+2}^{-} - d_{N+2}^{+} = 127{,}000$$

$$\frac{100}{\sum_{n=1}^{N} W_n} \times \sum_{n=1}^{N} (W_{n-1} - W_n) + d_{N+3}^{-} - d_{N+3}^{+} = 1,$$

$$W_{n-1} - W_n = \begin{cases} W_{n-1} - W_n, & \text{if } W_{n-1} \geq W_n \\ 0, & \text{if } W_{n-1} < W_n \end{cases}$$

$$\bar{d}^{-}, \bar{d}^{+} \geq 0, \quad d_i^{-} \cdot d_i^{+} = 0 \quad \forall i$$

and the production balance constraints are given by

$$I_n = I_{n-1} + P_n - Q_n, \quad n = 1, 2, \ldots, N$$

<u>Model 2:</u>

If in the Model 1 the goal of limiting the average employees laid-off is much more important than that of limiting the total cost, then the priority orders of P_3 and P_4 in the Model 1 shall be reversed, and the achievement function becomes:

$$\bar{a} = [P_1 (\sum_{n=1}^{N} d_n^{-}), P_2 (d_{N+1}^{+}), P_3 (d_{N+3}^{+}), P_4 (d_{N+2}^{+})]$$

and the other formulation is same as presented in Model 1.

3.4 Results and Discussion

Both models 1 and 2 are solved by using the following numerical data

$N = 5$

$Q_1 = 430$, $Q_2 = 447$, $Q_3 = 440$, $Q_4 = 316$, $Q_5 = 397$.

Initial inventory, $I_o = 263$

Initial work force, $W_o = 81$

Starting point:

$(P_1, P_2, P_3, P_4, P_5) = (400, 400, 400, 400, 400, 400)$

$(W_1, W_2, W_3, W_4, W_5) = (90, 90, 90, 90, 90)$

Models 1 and 2 are solved by using the iterative non-linear goal programming algorithm with the same starting point. The optimal solutions obtained from two models are presented in Tables 3.1 and 3.2. The optimal goal achievements are as follows.

Model 1:

$a_1^* = 0.0$, $a_2^* = 0.0$, $a_3^* = 1533.1$, $a_4^* = 4.1$

So,

(1) Priority P_1, is achieved, that is, absolute constraints are satisfied.

(2) Priority P_2, goal 1 is also satisfied, that is, there are no stock-outs.

(3) Priority P_3, goal 2, is not completely satisfied. The total cost is \$127,000 + 1533.1 = \$128,533.1 which is slightly above the budget of \$127,000.

(4) Priority P_4, goal 3, is not completely satisfied. The average percentage employees laid of is 1.0 + 4.1 = 5.1%. This is more than the allowed percentage of 1%.

Model 2:

$$a_1^* = 0.0, \; a_2^* = 0.0, \; a_3^* = 0.0, \; a_4^* = 7064.1$$

So,

(1) Priority, P_1, is achieved, that is, absolute constraints are satisfied.

(2) Priority P_2, goal 1, is also satisfied, that is, there are no stock-outs.

(3) Priority P_3, goal 2, is also satisfied. The average employees laid off is within 1%.

(4) Priority P_4, goal 3, is not satisfied. The total cost is \$127,000 + 7064.1 = \$134,064.1.

From models 1 and 2 we see that the cost is increased by \$5531.0 (134,064.1 - 128,533.1) to reduce the average employees laid off by 4.1% (5.1 - 1).

Table 3.1. Optimal results for model 1.

Month n	Demand Q_n	Production P_n	Work force W_n	End of period inventory I_n
0			81	263
1	430	461.3	76.8	294.3
2	447	425.5	72.6	272.8
3	440	381.4	68.6	214.2
4	316	356.8	65.1	255.0
5	397	347.1	63.3	205.1

Table 3.2. Optimal results for model 2.

Month n	Demand Q_n	Production P_n	Work force W_n	End of period inventory I_n
0			81	263
1	430	423.1	77.3	256.1
2	447	422.7	77.1	231.8
3	440	422.6	77.1	214.4
4	316	422.6	77.1	321.0
5	397	422.6	77.1	346.6

CHAPTER 4

A MULTIOBJECTIVE, MULTISTAGE, MULTIPRODUCT, SINGLE FACILITY, PRODUCTION PLANNING

Most firms manufacture a variety of products using a single facility instead of a single product. So aggregate production planning problems must deal with all products at the same time. The problem is further complicated when the problem is dynamic and the firm has multiple goals that are to be attained. In this chapter a general aggregate production planning problem with multiple objectives is first formulated as goal programming model and its application is explained through a numerical example.

Variables and Constants

b_j = Sum of the square of differences in production levels from period to period for the jth product.

C_i = Normal operating capacity of the plant during the ith period.

$\bar{d}^-$ = Vector of negative deviations from the desired goals.

$\bar{d}^+$ = Vector of positive deviations from the desired goals.

h_j = Number of hours required to produce one lb of the jth product.

I_j^i = Inventory of the jth product at the end of the ith period.

I_j^0 = Initial inventory of the jth product.

m = Number of products to be produced.

n = Number of periods in planning horizon.

O_i = Overtime operation of the plant during the ith period.

S_j^i = Demand for the jth product during the ith period.

x_j^i = Number of pounds of the jth product produced in the ith period.

Goals:

Let the following goal structure, in order of priority, represents the managements policy in the aggregate production.

(1) Achieve the sales goals for all products in each period.

(2) Limit the final inventory of the jth product at the end of planning period to q_j^n lbs.

(3) Avoid any underutilization of normal capacity in each period.

(4) Limit the sum of the squares of differences in production levels from period to period for the jth product to b_j.

(5) Limit overtime of operation of the plant to O_i in the ith period.

(6) Minimize final inventory of each product, at the end of the nth period, as much as possible.

The above goals can be represented mathematically as follows:

(1) Sales Goals:

The sum of initial inventory at the begining of any period and production during that period must meet the anticipated demand during that period. Mathematically this can be represented as

$$I_j^{i-1} + x_j^i \geq S_j^i , \quad i = 1,2, \ldots, m$$

we can rewrite the above goal incorporating deviation variables.

$$I_j^{i-1} + x_j^i + d^-_{n(j-1)+i} - d^+_{n(j-1)+i} = S_j^i , \quad \begin{array}{l} i = 1,2, \ldots, n \\ j = 1,2, \ldots, m \end{array}$$

where $d^-_{n(j-1)+i}$ and $d^+_{n(j-1)+i}$ represent shortage and closing inventory (excess production) of the jth product at the end of the period i, respectively. And also

$$I_j^i = d^+_{n(j-1)+i} , \quad i = 1,2, \ldots, n , \quad j = 1,2, \ldots, m.$$

(2) Desired final inventories:

The management desires to limit the final inventories at the end of planning horizon (I_j^n) to q_j^n units for each product. So

$$I_j^n \leq q_j^n , \quad j = 1,2, \ldots, m$$

introducing deviation variables to the above goal and rewriting

$$I_j^n + d^-_{(m.n+j)} - d^+_{(m.n+j)} = q_j^n , \quad j = 1,2, \ldots, m$$

where $d^-_{(m.n+j)}$ and $d^+_{(m.n+j)}$ represent the shortage and excess of the jth product from the desired level of final inventory q_j^n.

(3) Normal capacity of the plant:

The manager wishes to have no idle capacity, i.e., the required operating capacity must at least be equal to the available capacity. The required capacity in any period is equal to sum of hours required by each product's production during that period.

$$\sum_{j=1}^{m} h_j x_j^i \geq C_i , \quad i = 1,2, \ldots, n$$

or $$\sum_{j=1}^{m} h_j x_j^i + d^-_{m(1+n)+i} - d^+_{m(1+n)+i} = C_i , \quad i = 1,2, \ldots, n$$

where $d^-_{m(1+n)+i}$ and $d^+_{m(1+n)+i}$ represent underutilization and overutilization of production capacity respectively.

(4) Sum of the squares of differences in production levels from period to period for each product:

The manager wants to limit sum of the squares of differences in production levels from period to period to b_j.

$$\sum_{i=1}^{n-1} (x_j^{i+1} - x_j^i)^2 \leq b_j , \quad j = 1,2, \ldots, m$$

or $$\sum_{i=1}^{n-1} (x_j^{i+1} - x_j^i)^2 + d^-_{(n+m+mn+j)} - d^+_{(n+m+mn+j)} = b_j$$

$$j = 1,2, \ldots, m.$$

where $d^-_{n+m+mn+j}$ and $d^+_{n+m+mn+j}$ represent under achievement and over achievement from the desired goal for the jth product.

(5) Overtime operation of the plant:

The fifth goal is to limit the overtime operation of the plant to O_i hours in the ith period.

$$\sum_{j=1}^{m} h_j . x_j^i \leq C_i + O_i \,, \quad i = 1,2, \ldots, n$$

or rewriting

$$\sum_{j=1}^{m} (h_i . x_j^i) + d^-_{n+2m+mn+i} - d^+_{n+2m+mn+i} = C_i + O_i \,,$$

$$i = 1,2, \ldots, n.$$

where $d^-_{n+2m+mn+i}$ and $d^+_{n+2m+mn+i}$ represent underutilization and over utilization of overtime during the ith period.

(6) Final inventory of each product at the end of the nth period:

Finally, the manager wishes to minimize excess production, at the end of the nth period, as much as possible. That is, he does not want to produce any excess production by utilizing allowed overtime capacity. This can be achieved by minimizing the excess production over demand at the end of the nth period, I_j^n , $j = 1,2, \ldots, m$, at the last priority level.

Non-negative constraints:

$$\bar{d}^-, \bar{d}^+, \bar{x}, \bar{I} \geq 0$$

4.1 A General GP Model

The general model for aggregate production planning can now be formulated. The objective is the minimization of deviations from certain goals with assigned preemptive priority factors.

$$\text{Min } \bar{a} = \Big[\Big(\sum_{j=1}^{m} \sum_{i=1}^{n} d^-_{n(j-1)+i} \Big), \Big(\sum_{j=1}^{m} d^+_{mn+j} \Big), \Big(\sum_{i=1}^{n} d^-_{i+m+mn} \Big), \Big(\sum_{j=1}^{m} d^+_{n+m+mn+j} \Big), \Big(\sum_{i=1}^{n} d^+_{n+2m+mn+i} \Big), \Big(\sum_{j=1}^{m} d^+_{n(j-1)+n} \Big) \Big]$$

Subject to

$$I_j^{i-1} + x_j^i + d^-_{n(j-1)+i} - d^+_{n(j-1)+i} = S_j^i, \quad i = 1,2, \ldots, n; \quad j = 1,2, \ldots, m.$$

$$I_j^n + d^-_{mn+j} - d^+_{mn+j} = q_j^n, \quad j = 1,2, \ldots, m$$

$$\sum_{j=1}^{m} h_j x_j^i + d^-_{mn+m+i} - d^+_{mn+m+i} = C_i, \quad i = 1,2, \ldots, n$$

$$\sum_{i=1}^{n-1} (x_j^{i+1} - x_j^i)^2 + d^-_{n+m+mn+j} - d^+_{n+m+mn+j} = b_j, \quad j=1,2, \ldots, m$$

$$\sum_{j=1}^{m} (h_j x_j^i) + d^-_{n+2m+mn+i} - d^+_{n+2m+mn+i} = C_i + O_i, \quad i=1,2, \ldots, n$$

$$\bar{d}^-, \bar{d}^+, \bar{x}, \bar{I} \geq 0$$

The objective function indicates that the most important goal of management is to achieve the sales goals. Hence, the highest priority factor P_1 is assigned to the negative deviation from the demands.

Secondly the management desires to limit the final inventory (excess production) to q_j^n lbs. This is achieved by assigning priority factor P_2 to the minimization of positive deviations (d^+_{mn+j} , j=1,2, ..., m) from maximum desired final inventory. The production of final inventory is to absorb any underutilization of production capacity as far as possible without producing too much.

The third goal is to avoid underutilization of normal production capacity. In otherwords, the third goal is to keep employment as close to the level set in long range plans as possible. Therefore, P_3 is assigned to the negative deviations ($d^-_{m(1+n)+i}$, i=1,2, ..., n) from normal capacities (C_i).

The fourth goal is to limit the sum of squares of change in production levels from period to period for each product. In otherwards, the fourth goal is to have fairly a constant production in all periods. This avoids costs due to changes in production levels and to keep employment level fairly stable. Therefore, P_4 is assigned to the positive deviations ($d^+_{n+m+mn+j}$, j=1,2, ..., m) from allowable sum of squares of change in the production levels (b_j).

The fifth goal is to limit overtime operation of the plant to O_i , i=1,2, ..., n. This is achieved by minimizing the positive deviations ($d^+_{n+2m+mn+i}$, i=1,2, ..., n) from the permissible limit of overtime.

Finally, the last goal is to minimize the final inventories, (excess production). The management wishes to allow limited excess production only to utilize any unutilized normal production capacity. It does not want to produce more by utilizing overtime. This is achieved by minimizing positive deviation variables ($d^+_{n(j-1)+n}$, j=1,2, ..., m) at sixth priority level. This will eliminate any excess production produced utilizing overtime, retaining excess production, if any, produced utilizing idle capacity as we are minimizing underutilization of normal production capacity at a higher priority level.

The schematic representation of the problem is shown in Fig. 4.1.

4.2 Numerical Example

Assumption: Lost sales in any period can not be recovered.

Let n = Number of months in planning horigon = 3

m = Number of products = 2.

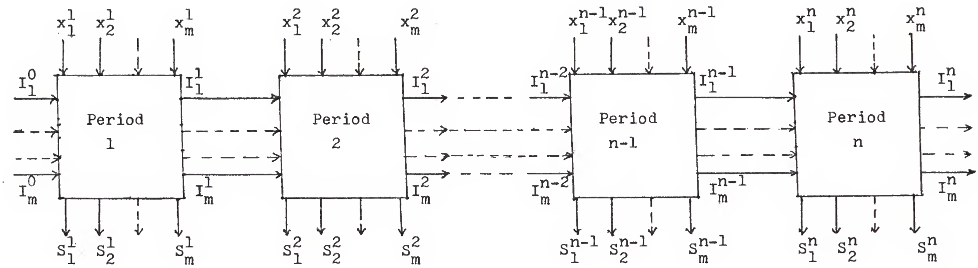

Fig. 4.1. Block diagram for aggregate production planning problem.

$S_1^1 = 45$ lbs., $S_2^1 = 60$ lbs.

$S_1^2 = 50$ lbs., $S_2^2 = 55$ lbs.

$S_1^3 = 55$ lbs., $S_2^3 = 65$ lbs.

$h_1 = 5$ hours, $h_2 = 6$ hours.

$I_1^0 = 0$, $I_2^0 = 0$

$b_1 = 30$, $b_2 = 30$

$q_1^3 = 10$ lbs., $q_2^3 = 10$ lbs.

$C_1 = 620$ hrs., $C_2 = 620$ hrs., $C_3 = 620$ hrs.

$O_1 = 30$ hrs., $O_2 = 30$ hrs., $O_3 = 30$ hrs.

<u>Goals</u>:

P_1 Achieve the sales goals for both products in each month.

P_2 Limit the final inventory (I_1^3 and I_2^3) of each product to 10 lbs.

P_3 Avoid any under utilization of production capacity in each period.

P_4 Limit the sum of squares of the difference in production levels from period to period for each product to 30.

P_5 Limit overtime operation of the plant to 30 hours in each period.

P_6 Minimize final inventories (excess production) of each product, at the end of final period, as far as possible.

(1) Sales Goals:

$$I_1^0 + x_1^1 + d_1^- - d_1^+ = S_1^1 = 45$$

$$I_1^1 + x_1^2 + d_2^- - d_2^+ = S_1^2 = 50$$

$$I_1^2 + x_1^3 + d_3^- - d_3^+ = S_1^3 = 55$$

$$I_2^0 + x_2^1 + d_4^- - d_4^+ = S_2^1 = 60$$

$$I_2^1 + x_2^2 + d_5^- - d_5^+ = S_2^2 = 55$$

$$I_2^2 + x_2^3 + d_6^- - d_6^+ = S_2^3 = 65$$

First priority of achieving sales goals can be obtained by minimizing $d_1^- + d_2^- + d_3^- + d_4^- + d_5^- + d_6^-$ at priority level 1.

(2) Final Inventory:

The second priority is assigned to limit the final inventory to 10 lbs. each.

$$I_1^3 + d_7^- - d_7^+ = q_1^3 = 10$$

$$I_2^3 + d_8^- - d_8^+ = q_2^3 = 10$$

The achievement function to minimized is: $a_2 = (d_7^+ + d_8^+)$.

(3) Normal Capacity:

$$5\,x_1^1 + 6\,x_2^1 + d_9^- - d_9^+ = 620 \qquad \text{(Period 1)}$$

$$5\,x_1^2 + 6\,x_2^2 + d_{10}^- - d_{10}^+ = 620 \qquad \text{(Period 2)}$$

$$5\,x_1^3 + 6\,x_2^3 + d_{11}^- - d_{11}^+ = 620 \qquad \text{(Period 3)}$$

The achievement function is: $a_3 = (d_9^- + d_{10}^- + d_{11}^-)$.

(4) Sum of squares of difference in production levels from period to period:

$$(x_1^2 - x_1^1)^2 + (x_1^3 - x_1^2)^2 + d_{12}^- - d_{12}^+ = b_1 = 30$$

$$(x_2^2 - x_2^1)^2 + (x_2^3 - x_2^2)^2 + d_{13}^- - d_{13}^+ = b_2 = 30$$

The corresponding achievement function is: $a_4 = (d_{12}^+ + d_{13}^+)$.

(5) Limit on overtime operation:

$$5\,x_1^1 + 6\,x_2^1 + d_{14}^- - d_{14}^+ = C_1 + O_1 = 620 + 30 = 650$$

$$5\,x_1^2 + 6\,x_2^2 + d_{15}^- - d_{15}^+ = C_2 + O_2 = 620 + 30 = 650$$

$$5\,x_1^3 + 6\,x_2^3 + d_{16}^- - d_{16}^+ = C_3 + O_3 = 620 + 30 = 650$$

The achievement function which is to be minimized is:

$$a_5 = (d_{14}^+ + d_{15}^+ + d_{16}^+).$$

Now the complete model can be formulated as below.

$$\text{Min } \bar{a} = \Big[(d_1^- + d_2^- + d_3^- + d_4^- + d_5^- + d_6^-),\ (d_7^+ + d_8^+),\ (d_9^- + d_{10}^- + d_{11}^-),$$
$$(d_{12}^+ + d_{13}^+),\ (d_{14}^+ + d_{15}^+ + d_{16}^+),\ (d_3^+ + d_6^+)\Big]$$

subject to

$$I_1^0 + x_1^1 + d_1^- - d_1^+ = 45$$

$$I_1^1 + x_1^2 + d_2^- - d_2^+ = 50$$

$$I_1^2 + x_1^3 + d_3^- - d_3^+ = 55$$

$$I_2^0 + x_2^1 + d_4^- - d_4^+ = 60$$

$$I_2^1 + x_2^2 + d_5^- - d_5^+ = 55$$

$$I_2^2 + x_2^3 + d_6^- - d_6^+ = 65$$

$$I_1^3 + d_7^- - d_7^+ = 10$$

$$I_2^3 + d_8^- - d_8^+ = 10$$

$$5\ x_1^1 + 6\ x_2^1 + d_9^- - d_9^+ = 620$$

$$5\ x_1^2 + 6\ x_2^2 + d_{10}^- - d_{10}^+ = 620$$

$$5\ x_1^3 + 6\ x_2^3 + d_{11}^- - d_{11}^+ = 620$$

$$(x_1^2 - x_1^1)^2 + (x_1^3 - x_1^2)^2 + d_{12}^- - d_{12}^+ = 30$$

$$(x_2^2 - x_2^1)^2 + (x_2^3 - x_2^2)^2 + d_{13}^- - d_{13}^+ = 30$$

$$5\ x_1^1 + 6\ x_2^1 + d_{14}^- - d_{14}^+ = 650$$

$$5\ x_1^2 + 6\ x_2^2 + d_{15}^- - d_{15}^+ = 650$$

$$5\ x_1^3 + 6\ x_2^3 + d_{16}^- - d_{16}^+ = 650$$

$$\bar{d}^- ,\ \bar{d}^+ ,\ \bar{x} \geq 0$$

The above problem is solved with an initial point as (0,0,0,0,0,0) by computer using the iterative nonlinear goal programming package. The results are tabulated in tables 4.1 a to 4.1 f.

4.3 Results and Discussion

Our first priority is to achieve sales goals. At the initial starting point (0,0,0,0,0,0), it is clear that goal 1 is not attained. Total lost sales is 330 lbs.

Any point which lies in the solution space ($45 \leq x_1^1 < \infty$, $60 \leq x_2^1 < \infty$, $50 \leq x_1^2 < \infty$, $55 \leq x_2^2 < \infty$, $55 \leq x_1^3 < \infty$, $65 \leq x_2^3 < \infty$ satisfies goal 1. The problem is solved by computer using iterative nonlinear goal programming algorithum and the results are tabulated in table 4.1 a. From the table, we see that one such point is (65,65,65,65,65,65) where goal 1 is completely attained.

Our next priority is to limit the final inventories (excess production) of both products to 10 lbs each. SO our solution should lie in the common solution space I intersection of ($45 \leq x_1^1 < \infty$, $60 \leq x_2^1 < \infty$, $50 \leq x_1^2 < \infty$, $55 \leq x_2^2 < \infty$, $55 \leq x_1^3 < \infty$, $65 \leq x_2^3 < \infty$) and ($0 \leq I_1^3 \leq 10$, $0 \leq I_2^3 \leq 10$).

At our previous point (65,65,65,65,65,65) which satisfies goal 1 lies out side this common solution space I. We have values of $I_1^3 = 45$ and $I_2^3 = 15$. In other words we have $d_7^+ = 35$ (45-10) and $d_8^+ = 5$ (10-5). So in order to satisfy goal 1 and 2, we should find a point within the common solution space I. This is achieved by minimizing d_7^+ and d_8^+

at priority level 2. The results are tabulated in table 4.1 b. The point (51,69,51,61,51,59) obtained, lies within this common solution space I. So both goals 1 and 2 are satisfied. We have $I_1^3 = 3$ and $I_2^3 = 9$, which are within the limits specified.

Our third goal is to minimize underutilization of production capacity. So our solution should lie in the common solution space II, intersection of ($45 \leq x_1^1 < \infty$, $60 \leq x_2^1 < \infty$, $50 \leq x_1^2 < \infty$, $55 \leq x_2^2 < \infty$, $55 \leq x_1^3 < \infty$, $65 \leq x_2^3 < \infty$), ($0 \leq I_1^3 \leq 10$, $0 \leq I_2^3 \leq 10$), and ($5\,x_1^1 + 6\,x_2^1 \geq 620$, $5\,x_2^2 + 6\,x_2^2 \geq 620$, $5\,x_1^3 + 6\,x_2^3 \geq 620$). The previous point (51,69,51,61,51,59) does not lie in this common solution space II, as we have, from the previous results, table 4.1 b, $5\,x_1^3 + 6\,x_2^3 = 609$ which is less by 11 hours than the desired value of 620 hours. So in order to satisfy third priority, we should move from our previous point to the feasible common solution space II. The problem is solved by minimizing negative deviations ($d_9^- + d_{10}^- + d_{11}^-$) from goals, with previous point as starting point, using iterative nonlinear goal program. The results are presented in table 4.1 c. The point (52,70,52,60,52,60) obtained lies in the above common solution space and so it satisfies all the three goals.

The fourth goal is to limit each sum of squares of changes in production levels from period to period to 30. So our solution should lie in the common solution space III, the intersection of ($45 \leq x_1^1 < \infty$, $60 \leq x_2^1 < \infty$, $50 \leq x_1^2 < \infty$, $55 \leq x_2^2 < \infty$, $55 \leq x_1^3 < \infty$, $65 \leq x_2^3 < \infty$), ($0 \leq I_1^3 \leq 10$, $0 \leq I_2^3 \leq 10$), ($5\,x_1^1 + 6\,x_2^1 \geq 620$, $5\,x_1^2 + 6\,x_2^2 \geq 620$, $5\,x_1^3 + 6\,x_2^3 \geq 620$), and [$(x_1^2 - x_1^1)^2 + x_1^3 - x_1^2)^2 \leq 30$, $(x_2^2 - x_2^1)^2 + (x_2^3 - x_2^2)^2 \leq 30$].

The previous point (52,70,52,60,52,60) does not lie within this common solution space III, because at this point we have $(x_2^2 - x_2^1)^2 + (x_2^3 - x_2^2)^2 = 100$, which exceeds by 70 (=100-30) than the maximum permissible limit of 30. The minimization of this excess is obtained by minimizing (d_{12}^+ and d_{13}^+) at fourth priority level, so that the solution also satisfies the previously attained goals. So with previous point (52,70,52,60,52,60) as starting point, the problem is solved to satisfy fourth goal and also the first three goals using nonlinear goal programming algorithum. The results are tabulated in Table 4.1 d. We have obtained a point (53.25, 67.25, 54.5, 62.5, 52, 60) which lies in the common solution space and satisfies all goals.

The fifth goal is to limit the overtime utilized to 30 hours in each period. To satisfy this goal, the solution

should lie in the common solution space IV, the intersection of $(45 \leq x_1^1 < \infty$, $60 \leq x_2^1 < \infty$, $50 \leq x_1^2 < \infty$, $55 \leq x_2^2 < \infty$, $55 \leq x_1^3 < \infty$, $65 \leq x_2^3 < \infty)$, $(0 \leq I_1^3 \leq 10$, $0 \leq I_2^3 \leq 10)$, $(650 \geq 5\ x_1^1 + 6\ x_2^1 \geq 620$, $650 \geq 5\ x_1^2 + 6\ x_2^2 \geq 620$, $650 \geq 5\ x_1^3 + 6\ x_2^3 \geq 620)$, and $[(x_1^2 - x_1^1)^2 + (x_1^3 - x_1^2)^2 \leq 30$, $(x_2^2 - x_2^1)^2 + (x_2^3 - x_2^2)^2 \leq 30\ ]$. The previous point (53, 25, 67.25, 54.5, 62.5, 52, 60), which satisfied the first four goals, does not lie in the present common solution space IV, because $5\ x_1^1 + 6\ x_2^1 = 669.8$, which exceeds by 19.8 hours more than the desired value of 650. So the present point should be moved into the common solution space IV in order to satisfy all the five goals. This is achieved by minimizing positive deviations from the allowable overtime at fifth priority level without sacrificing the first four goals that are satisfied. The problem is solved with the previous point (53.25, 67.25, 54.5, 62.5, 52, 60) as starting point, using iterative nonlinear goal program, and the results are tabulated in table 4.1 e. From the results we see that, the point (51.25, 65.25, 52.5, 64.5, 54, 60) lies in the solution space IV and satisfies all the five goals.

Our final goal, the sixth goal, is to minimize final inventories (excess production) as far as possible. To satisfy this goal the solution should lie in the common solution space V, the

intersection of $(45 \leq x_1^1 < \infty,\ 60 \leq x_2^1 < \infty,\ 50 \leq x_1^2 < \infty,$ $55 \leq x_2^2 < \infty,\ 55 \leq x_1^3 < \infty,\ 65 \leq x_2^3 < \infty),\ (0 \leq I_1^3 \leq 10,$ $0 \leq I_2^3 \leq 10),\ (650 \geq 5\, x_1^1 + 6\, x_2^1 \geq 620,\ 650 \geq 5\, x_1^2 + 6\, x_2^2 \geq 620,$ $650 \geq 5\, x_1^3 + 6\, x_2^3 \geq 620),\ [(x_1^2 - x_1^1)^2 + (x_1^3 - x_1^2)^2 \leq 30,$ $(x_2^2 - x_2^1)^2 + (x_2^3 - x_2^2)^2 \leq 30]$, and $(I_1^3 = 0,\ I_2^3 = 0)$.

Our previous point (51.25, 65.25, 52.5, 64.5, 54, 60) which satisfied the first five goals does not lie in the solution space V, as it is not satisfying the sixth goal. From table 4.1 e, we see that $I_1^3 = 7.75$, $I_2^3 = 9.75$. In order to achieve the sixth goal as far as possible, d_3^+ and d_6^+ are minimized at sixth priority level and the results are tabulated in table 4.1 f. At the point (48.1, 63.25, 50.2, 61.5, 52, 60), the first five goals are completely satisfied, but the sixth goal is not completely attained. We have $I_1^3 = 0.3$, $I_2^3 = 4.75$. If we try to reduce these values further, we increase the underutilization of production capacity, which is now 0 hours, which is highly undesirable because it is at higher priority level. So, the best compromisable solution for the problem is to follow the production schedule (48.1, 63.25, 50.2, 61.5, 52, 60) which satisfies the first five goals.

Table 4.1 a. Results after achieving Priority level 1.

	Initial Inventory of Product		Production of Product		Demand of Product		Lost Sales of Product		End of Period Inv. of Product		Change in Production of Product		Sq. of Change in Production of Product		Regular capacity			Overtime capacity			Excess Production of Product	
	1 I_1^1	2 I_2^1	1 x_1^1	2 x_2^1	1 S_1^1	2 S_2^1	1	2	1 I_1^1	2 I_2^1	1	2	1	2	Available	Utilized	Un-uti.	Available	Utiliz.	Over uti.	1	2
Period i = 1	0.0	0.0	65.0 (0.0)*	65.0 (0.0)	45.0	60.0	0.0 (45.0)	0.0 (60.0)	20.0	5.0	-	-	-	-	620.0	715.0	0.0	30.0	95.0	65.0		
Period i = 2	20.0	5.0	65.0 (0.0)	65.0 (0.0)	50.0	55.0	0.0 (50.0)	0.0 (55.0)	35.0	15.0	0.0	0.0	0.0	0.0	620.0	715.0	0.0	30.0	95.0	65.0		
Period i = 3	35.0	15.0	65.0 (0.0)	65.0 (0.0)	55.0	65.0	0.0 (50.0)	0.0 (65.0)	45.0	15.0	0.0	0.0	0.0	0.0	620.0	715.0	0.0	30.0	95.0	65.0	45.0	15.0
a_1							0.0 (330.0)															
a_2									40.0													
a_3																	0.0					
a_4													0.0									
a_5																				195.0		
a_6																					60.0	

* Values in parenthesis indicate the values at the starting point.

Table 4.1 b. Results after achieving Priority level 2.

	Initial Inventory of Product 1 I_1^1	Initial Inventory of Product 2 I_2^1	Production of Product 1 x_1^1	Production of Product 2 x_2^1	Demand of Product 1 S_1^1	Demand of Product 2 S_2^1	Lost Sales of Product 1	Lost Sales of Product 2	End of Period Inv. of Product 1 I_1^1	End of Period Inv. of Product 2 I_2^1	Change in Production of Product 1	Change in Production of Product 2	Sq. of Change in Production of Product 1	Sq. of Change in Production of Product 2	Regular capacity Avai-lable	Regular capacity Util-ized	Regular capacity Un-uti.	Overtime capacity Avai-lable	Overtime capacity Uti-liz.	Overtime capacity Over uti.	Excess Production of Product 1	Excess Production of Product 2
Period 1 = 1	0.0	0.0	51.0 (65.0)*	69.0 (65.0)	45.0	60.0	0.0	0.0	6.0	9.0	-	-	-	-	620.0	669.0	0.0	30.0	49.0	19.0		
Period 1 = 2	6.0	9.0	51.0 (65.0)	61.0 (65.0)	50.0	55.0	0.0	0.0	7.0	15.0	0.0	8.0	0.0	64.0	620.0	621.0	0.0	30.0	1.0	0.0		
Period 1 = 3	7.0	15.0	51.0 (65.0)	59.0 (65.0)	55.0	65.0	0.0	0.0	3.0 (45.0)	9.0 (15.0)	0.0	2.0	0.0	4.0	620.0	609.0	11.0	30.0	0.0	0.0	3.0 (45.0)	9.0 (15.0)
a_1							0.0															
a_2									0.0 (40.0)													
a_3																	11.0					
a_4													38.0									
a_5																				19.0		
a_6																						12.0 (60.0)

* Values in parenthesis indicate the values at the starting point.

Table 4.1 c. Results after achieving Priority level 3.

	Initial Inventory of Product		Production of Product		Demand of Product		Lost Sales of Product		End of Period Inv. of Product		Change in Production of Product		Sq. of Change in Production of Product		Regular capacity			Overtime capacity			Excess Production of Product	
	1 I_1^1	2 I_2^1	1 x_1^1	2 x_2^1	1 s_1^1	2 s_2^1	1	2	1 I_1^1	2 I_2^1	1	2	1	2	Available	Utilized	Un-uti.	Available	Utiliz.	Over utl.	1	2
Period 1 = 1	0.0	0.0	52.0 (51.0)*	70.0 (69.0)	45.0	60.0	0.0	0.0	7.0	10.0	-	-	-	-	620.0	680.0 (669.0)	0.0	30.0	60.0	30.0		
Period 1 = 2	7.0	10.0	52.0 (51.0)	60.0 (61.0)	50.0	55.0	0.0	0.0	9.0	15.0	0.0	10.0	0.0	100.0	620.0	620.0 (621.0)	0.0	30.0	0.0	0.0		
Period 1 = 3	9.0	15.0	52.0 (51.0)	60.0 (59.0)	55.0	65.0	0.0	0.0	6.0	10.0	0.0	0.0	0.0	0.0	620.0	620.0 (609.0)	0.0 (11.0)	30.0	0.0	0.0	6.0	10.0
a_1							0.0															
a_2									0.0													
a_3																	0.0 (11.0)					
a_4													70.0									
a_5																				30.0		
a_6																					16.0	

* Values in parenthesis indicate the values at the starting point.

Table 4.1 d. Results after achieving Priority level 4.

	Initial Inventory of Product 1 I_1^1	Initial Inventory of Product 2 I_2^1	Production of Product 1 x_1^1	Production of Product 2 x_2^1	Demand of Product 1 S_1^1	Demand of Product 2 S_2^1	Lost Sales of Product 1	Lost Sales of Product 2	End of Period Inv. of Product 1 I_1^1	End of Period Inv. of Product 2 I_2^1	Change in Production of Product 1	Change in Production of Product 2	Sq. of Change in Production of Product 1	Sq. of Change in Production of Product 2	Regular capacity Available	Regular capacity Utilized	Regular capacity Un-uti.	Overtime capacity Available	Overtime capacity Utiliz.	Overtime capacity Over uti.	Excess Period Inv. of Product 1	Excess Period Inv. of Product 2
Period i = 1	0.0	0.0	53.25 (52.0)*	67.25 (70.0)	45.0	60.0	0.0	0.0	8.25	7.25	-	-	-	-	620.0	669.8	0.0	30.0	49.8	19.8		
Period i = 2	8.25	7.25	54.5 (52.0)	62.5 (60.0)	50.0	55.0	0.0	0.0	12.75	14.75	1.25	4.75	1.6 (0.0)	22.6 (100.0)	620.0	647.5	0.0	30.0	27.5	0.0		
Period i = 3	12.75	14.75	52.0 (52.0)	60.0 (60.0)	55.0	65.0	0.0	0.0	9.75	9.75	2.5	2.5	6.25	6.25	620.0	620.0	0.0	30.0	0.0	0.0	9.75	9.75
a_1							0.0															
a_2									0.0													
a_3																	0.0					
a_4													0.0 (70.0)									
a_5																				19.8		
a_6																					19.5	

* Values in parenthesis indicate the values at the starting point.

Table 4.1 e. Results after achieving Priority level 5.

	Initial Inventory of Product 1 I_1^1	Initial Inventory of Product 2 I_2^1	Production of Product 1 x_1^1	Production of Product 2 x_2^1	Demand of Product 1 S_1^1	Demand of Product 2 S_2^1	Lost Sales of Product 1	Lost Sales of Product 2	End of Period Inv. of Product 1 I_1^1	End of Period Inv. of Product 2 I_2^1	Change in Production of Product 1	Change in Production of Product 2	Sq. of Change in Production of Product 1	Sq. of Change in Production of Product 2	Regular capacity Avai-lable	Regular capacity Util-ized	Regular capacity Un-uti.	Overtime capacity Avai-lable	Overtime capacity Uti-liz.	Overtime capacity Over uti.	Excess Production of Product 1	Excess Production of Product 2
Period 1 = 1	0.0	0.0	51.25 (53.25)*	65.25 (67.25)	45.0	60.0	0.0	0.0	6.25	5.25	-	-	-	-	620.0	647.75	0.0	30.0	27.75 (49.8)	0.0		
Period 1 = 2	6.25	5.25	52.5 (54.5)	64.5 (62.5)	50.0	55.0	0.0	0.0	8.75	14.75	1.25	0.75	1.56	0.56	620.0	649.5	0.0	30.0	29.5 (27.75)	0.0		
Period 1 = 3	8.75	14.75	54.0 (52.0)	60.0 (60.0)	55.0	65.0	0.0	0.0	7.75	9.75	1.5	4.5	2.25	20.25	620.0	630.0	0.0	30.0	10.0 (0.0)	0.0	7.75	9.75
a_1							0.0															
a_2									0.0													
a_3																	0.0					
a_4													0.0									
a_5																				0.0 (19.8)		
a_6																					17.5	

* Values in parenthesis indicate the values at the starting point.

Table 4.1 f. Results after achieving Priority level 6.

	Initial Inventory of Product		Production of Product		Demand of Product		Lost Sales of Product		End of Period Inv. of Product		Change in Production of Product		Sq. of Change in Production of Product		Regular capacity			Overtime capacity			Excess Production of Product	
	1 I_1^1	2 I_2^1	1 x_1^1	2 x_2^1	1 S_1^1	2 S_2^1	1	2	1 I_1^1	2 I_2^1	1	2	1	2	Available	Utilized	Un-uti.	Available	Utiliz.	Over uti.	1	2
Period i = 1	0.0	0.0	48.1 (51.25)*	63.25 (65.25)	45.0	60.0	0.0	0.0	3.1	3.25	-	-	-	-	620.0	620.0	0.0	30.0	0.0	0.0		
Period i = 2	3.1	3.25	50.2 (52.5)	61.5 (64.5)	50.0	55.0	0.0	0.0	3.3	9.75	2.1	1.75	4.41	3.06	620.0	620.0	0.0	30.0	0.0	0.0		
Period i = 3	3.3	9.75	52.0 (54.0)	60.0 (60.0)	55.0	65.0	0.0	0.0	0.3	4.75	1.8	1.5	3.24	2.25	620.0	620.0	0.0	30.0	0.0	0.0	0.3 (7.75)	4.75 (9.75)
a_1							0.0															
a_2									0.0													
a_3																	0.0					
a_4													0.0									
a_5																				0.0		
a_6																						5.05 (17.5)

* Values in parenthesis indicate the values at the starting point.

CHAPTER 5

CONCLUSIONS

Multiple criteria decision making through goal programming has been performed through the use of nonlinear goal programming. An algorithm is developed by modifying the Hooke and Jeeves pattern search technique to solve the nonlinear goal programming problems iteratively. In this technique, lower priority goals are considered only after the higher priority goals are satisfied or have reached the point beyond which no further improvements are possible. The new technique allows us to solve many of the applied nonlinear multiple objective problems that exist.

The capability of the iterative nonlinear goal programming technique is shown by applying it to aggregate production planning problems where multiple objectives exist. The model in this thesis considers only some of the common objectives found in aggregate production planning. It is, of course possible to develop models even further in many respects. For example, in the general model of Chapter 4, use of subcontracting may be added as an additional objective to the problem.

There is much scope for improvement in present techniques of solving nonlinear goal programming problems. Sequential simplex method may be modified to solve NLGP problems iteratively. Other possible areas are nonlinear integer goal programming for stochastic systems, and geometric programming.

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ITERATIVE NONLINEAR GOAL PROGRAMMING AND APPLICATION TO PRODUCTION PLANNING PROBLEMS

by

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ABSTRACT

Multiple conflicting objectives exist in most real world problems. So modern decision analysis must deal with all these conflicting objectives. Goal programming seems to be an appropriate technique for solving decision problems with multiple conflicting objectives. Decision problems become more complex when these objectives are nonlinear in nature.

A new algorithm, which integrates the iterative approach and modified Hook and Jeeves pattern search, is developed and the solution procedure is explained through a numerical example.

Next, nonlinear goal programming is applied to aggregate production planning problems. Holt's model for production planning is modified by adding two more objectives and solved using nonlinear goal programming. Also, a general multi-objective aggregate production problem is formulated as a goal programming model and the solution is obtained by using nonlinear goal programming approach.